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Orbifolds vs smooth manifolds with fluxes
reconciliating two different approaches to string phenomenology

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Based on
hep-th/0612030 (JHEP0701) with G. Honecker
hep-th/0701227 (JHEP07??) with S. Groot Nibbelink and M. Walter

Motivations I: The String Phenomenology “Paradigm”

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- Low energy (Heterotic) string theory

$\Rightarrow$ $d = 10, \mathcal{N} = 1$ SUGRA, $SO(32)$ SYM.

- Necessary a compactification on an “internal space” $\mathbb{M}^{10} \to \mathbb{M}^4 \times K^6$,

$\Rightarrow$ such that SUSY is reduced to $d = 4\ \mathcal{N} = 1$,

$\Rightarrow$ and the gauge symmetry is also reduced.

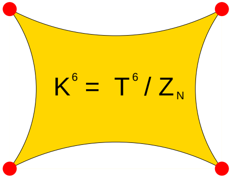
- More in general, we have to select a *background* for all the fields that are scalars of the 4d Minkowski group (internal components of gauge bosons, 2-forms, etc.):

$$A^M \longrightarrow (A^\mu, A^i) \;\Rightarrow\; \langle A^i \rangle \neq 0.$$

$\Rightarrow$ Flux compactification:
for us now mainly gauge fluxes.

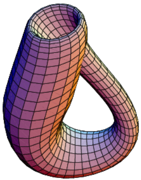
Motivations II: Which background?

- Toroidal orbifold



$\Rightarrow$ Exact string quantization.
$\Rightarrow$ Complete control on the spectrum of the model.
$\Rightarrow$ Bad control of the (twisted field) lagrangian.
$\Rightarrow$ No control on the potential of the scalar fields.

- Smooth manifold



$\Rightarrow$ No string quantization.
$\Rightarrow$ Only chiral spectrum known.
$\Rightarrow$ "Controlled" stabilization of scalars through (closed string) fluxes.

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Reconciliating the two approaches

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Each model built via an orbifold compactification of heterotic string has a counterpart, built via a compactification on a smooth manifold (in the presence of $U(1)$ gauge fluxes).

Outline

I Review of heterotic string orbifolds.

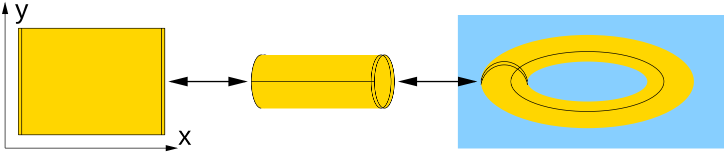
II String models on smooth spaces with $U(1)$ fluxes.

III Merging the approaches:

- Formal matching: the T^4/Z_2 models vs $K3$ models.
- An explicit example: Heterotic string on the blow-up of one of the T^6/Z_3 singularities vs heterotic string on T^6/Z_3.

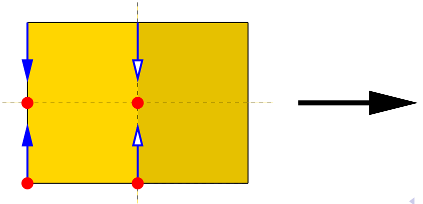
What's a (toroidal) orbifold?
A two dimensional example: T^2/Z_2

- Define T^2 as "a piece of complex plane" with parameter z.



$x \sim x + n$, $y \sim y + m$ $\longrightarrow$ $z = x + i\,y$ $\longrightarrow$ $z \sim z + n + i\,m$

- Define the Z_2 orbifold action on z: $z \to -z$ and identify the torus under such an action.



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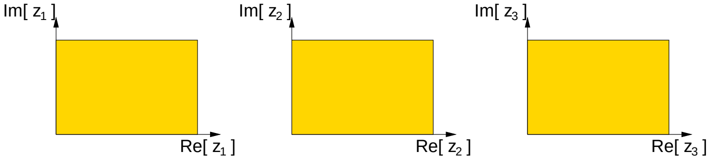
The general case: T^6/Z_N

- Define T^6 as three copies of the previous T^2, with parameters z_i.



- Define the Z_N orbifold action on z_i:

$$z \longrightarrow e^{2\pi i \frac{v_i}{N}} z_i$$

and identify T^6 under such an action.

$\Rightarrow$ All the informations in the vector $v = (v_1, v_2, v_3)$.

The orbifold operator acts in the gauge bundle too! The $SO(32)$ case.

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- **The gauge embedding ...**
  - Let T be a generator of $SO(32)$.
  - The orbifold action on it is,

$$T \longrightarrow \gamma_{Z_N} T \gamma_{Z_N}^{-1}.$$

  - Define H_I as the 16 elements of the Cartan subalgebra, and let the other generator be E_ω, such that $[H_I, E_\omega] = \omega_I E_\omega$.
  - Since Z_N is abelian, $\gamma_{Z_N} = e^{2\pi i \frac{V^I}{N} H_I}$.

  $\Rightarrow$ All the informations in the vector $V^I = (V^1, \ldots, V^{16})$.
- **... determines the gauge symmetry breaking!**
  - Under the orbifold action:

$$H_I \longrightarrow H_I, \quad E_\omega \longrightarrow E_\omega e^{2\pi i \frac{V^I \omega_I}{N}}.$$

  $\Rightarrow$ Rank preserving gauge symmetry breaking: All the H_I are "kept", the E_ω with non trivial phase are projected away.

Consistency conditions in a Z_N orbifold

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- **Z_N orbifold action:**

$$z_i \longrightarrow e^{2\pi i \frac{v_i}{N}} z_i \quad \Rightarrow \quad v_i = integer \ \ \forall\, i$$

$$T \longrightarrow \gamma_{Z_N} T \gamma_{Z_N}^{-1} \quad \Rightarrow \quad V^I = integer\ \forall I$$

$$\text{or} \quad V^I = half - integer\ \forall I.$$

- **SUSY:**

$$\sum_{i=1}^{3} v_i = even.$$

- **Modular invariance of the string partition function:**

$$\frac{1}{N}\left(\sum v_i^2 - \sum V^{I^2}\right) = even$$

Dixon, Harvey, Vafa, Witten, **NPB 261 (1985)**

Ibanez, Nilles, Quevedo, . . . '86 - '90

Kobayashi, Raby, Zhang, '04; Hebecker, M.T., '04; Buchmüller, Hamaguchi, Lebedev, Ratz, '04; Förste, Nilles, Vaudrevange, Wingerter, '04.

Consistent T^6/Z_3 models

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- In the Z_3 case only the following V^I's (up to equivalence) fulfil the requirements, with the given gauge symmetry breaking

Giedt, hep-th/0301232; Choi, Groot Nibbelink, M.T., hep-th/0410232.

(0^{16})	$SO(32)$	$(0^{13}, 1^2, 2)$	$SO(26) \times U(3)$
$(0^{10}, 1^4, 2^2)$	$SO(20) \times U(6)$	$(0^7, 1^6, 2^3)$	$SO(14) \times U(9)$
$(0^4, 1^8, 2^4)$	$SO(8) \times U(12)$	$(0^1, 1^{10}, 2^5)$	$SO(2) \times U(15)$

- This means that the local structure of each singularity is completely determined by one out of 6 possible V^I.
- Nevertheless, we can have different gauge embeddings V^I in each of the 27 T^6/Z_3 singularities, that are all equivalent from a purely geometrical perspective (Discrete Wilson lines).

Smooth manifolds with fluxes: guiding principle

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1. Given the low-energy spectrum and lagrangian of Heterotic string on 10d space

$$S \sim \int_{\mathbb{M}^{10}} d^{10}x \sqrt{g_{10}}\, e^{-2\phi} \left[R + (\partial\phi)^2 - |H_3|^2 - F^2\right] = \int_{\mathbb{M}^{10}} d^{10}x \sqrt{g_{10}}\, \mathcal{L}_{10},$$

2. we can choose a suitable smooth internal space K^6 and define a 4d lagrangian and spectrum via the Kaluza-Klein reduction:

$$S = \int_{\mathbb{M}^4 \times K^6} d^4x\, d^6y \sqrt{g_4\, g_6}\, \mathcal{L}_{10} = \int_{\mathbb{M}^4} d^4x \sqrt{g_4}\, \mathcal{L}_4.$$

   Also the internal components of F and H can be taken to be non-trivial, but let us reduce to the $\langle H \rangle = 0$ case.

3. **The chiral spectrum can be computed by checking the reduction of the 10d anomaly polynomial, i.e. via the Dirac index.**

! The approach is valid only if the low energy 10d SUGRA description of string theory is valid, i.e. if the "typical" lengths of the internal space are *much larger* than α'.

Which manifolds? Which fluxes? Constraints.

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- **SUSY and Background e.o.m:**
  - I K^6 must be Ricci flat.
  - II The F-flux must satisfy the Yang-Mills field equations.
  - III The e.o.m. and Bianchi Identity for H must be satisfied

$$H = dB - \frac{\alpha'}{4}(\omega_3^{YM} - \omega_3^{G}) \quad \Rightarrow \quad dH = \frac{\alpha'}{4}(F \wedge F - R \wedge R)$$

$$\Rightarrow \quad \int_{\gamma_4} (F \wedge F - R \wedge R) \; = \; \int_{\gamma_4} dH = \; 0.$$

- **The F-flux must be quantized.**

Smooth space compactifications:
Fradkin, Tseytlin **PLB 158 (1985)**; Candelas, Horowitz, Strominger, Witten, **NPB 258 (1985)** Strominger, **NPB 274 (1986)**; Abouelsaood, Callan, Nappi, Yost, **NPB 280 (1987)**.

Flux & SUSY breaking (open string orbifolds):
Bachas, hep-th/9503030; Bianchi, Stanev, hep-th/9711069; Angelantonj, Antoniadis, Dudas, Sagnotti, hep-th/0007090; Larosa, Pradisi, hep-th/0305224

Flux & realistic Heterotic models: Donagi, Lukas, Ovrut, Waldram, hep-th/9811168; Andreas, Curio, Klemm, hep-th/9903052; Bouchard, Donagi, hep-th/0512149.

U(1) fluxes on smooth backgrounds: Blumehagen, Honecker '05

Gauge symmetry breaking

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- Consider a complex two form $\mathcal{F}$ defined in the internal space, quantized and satisfying the Yang-Mills equations:

  it is a good $U(1)$ flux.

- Such a flux can be embedded in the $SO(32)$ gauge group in many different ways

$$F = \bar{V}^I H^I \mathcal{F}$$

- The gauge group is broken, due to such a flux, to the subgroup of $SO(32)$ that commutes with F, i.e.

$$\text{all the } H_I; \qquad \text{the } E_\omega \text{ such that } \left[E_\omega, \bar{V}^I H^I\right] = 0$$

  Similar to the unbroken gauge group in the orbifold case.

A formal example: the $K3$ models with flux as realization of T^4/Z_2 orbifold models

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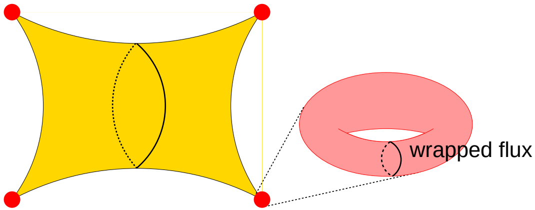
Honecker, M.T., hep-th/0612030

- Given $K3$ we know that there are 20 $(1,1)$-cycles.
- T^4/Z_2 is an orbifold limit of K3, in the orbifold perspective 4 of the 20 cycles are living on the original T^4 (untwisted), the other 16 are localized each in one of the 16 orbifold singularities (twisted).



The effect of V in the orbifold is seen in the smooth case as a gauge flux $\mathcal{F}$ wrapped on the "localized" cycle.

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- We want to "mimic" a T^4/Z_2 orbifold model by using fluxes on $K3$

$\Rightarrow$ Wrap a flux $F = \bar{V}^I H^I \mathcal{F}$ on each of the 16 "twisted" cycles, and nothing on the "untwisted" ones.

- For simplicity we want to avoid the issue of discrete Wilson lines

$\Rightarrow$ Wrap *the same* flux on each of the 16 "twisted" cycles.

- From the integrated Bianchi Identity we get the condition $\bar{V}^2 = 6$, that allows us to reconstruct all (and only) the known T^4/Z_2 models.

Matching the classifications: T^4/Z_2 orbifold models vs fluxed $K3$ models

Orbifold models: $V^2 = 2 \bmod 4$	$K3$ **models:** $\bar{V}^2 = 6$
$(1^2, 0^{14}) \Rightarrow SO(28) \times SU(2) \times SU(2)$ $(\mathbf{28}, \mathbf{2}, \mathbf{2}) + 4(\mathbf{1}, \mathbf{1}, \mathbf{1}) +$ $8(\mathbf{28}, \mathbf{1}, \mathbf{2}) + 32(\mathbf{1}, \mathbf{2}, \mathbf{1})$	$(1^2, 2, 0^{13}) \Rightarrow SO(26) \times U(1) \times U(2)$ $2(\mathbf{26}, \mathbf{2}) + 14(\mathbf{26}, \mathbf{1})$ $+36(\mathbf{1}, \mathbf{2}) + 34(\mathbf{1}, \mathbf{1})$
$(1^6, 0^{10}) \Rightarrow SO(20) \times SO(12)$ $(\mathbf{20}, \mathbf{12}) + 4(\mathbf{1}, \mathbf{1}) + 8(\mathbf{1}, \mathbf{32}_+)$	$(1^6, 0^{10}) \Rightarrow SO(20) \times U(6)$ $2(\mathbf{20}, \mathbf{6}) + 14(\mathbf{1}, \mathbf{15}) + 20(\mathbf{1}, \mathbf{1})$
$\frac{1}{2}(1^{15}, -3) \Rightarrow U(16)$ $2(\mathbf{120}) + 4(\mathbf{1}) + 16(\mathbf{16})$	$\frac{1}{2}(1^{15}, -3) \Rightarrow U(15) \times U(1)$ $2(\mathbf{105}) + 20(\mathbf{1}) + 16(\mathbf{15})$

An explicit example: blow-up of the T^6/Z_3 singularities

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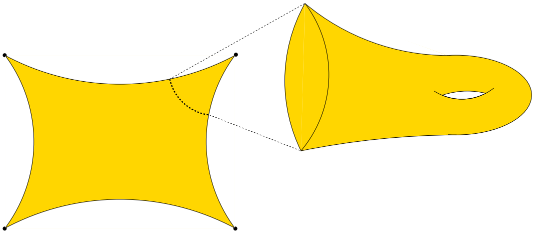
Groot Nibbelink, M.T., Walter, hep-th/0701227

- Take the original orbifold, "cut apart" one of the singularities, blow it up:

⇒ Get a smooth (non-compact) space

⇒ Use it as internal K^6 space in the compactification with gauge fluxes



- The whole smooth manifold will be given by patching the blow-up singularities.

- In the T^6/Z_3 case there are 27 equivalent singularities.
- We can cut one of them and consider its blow-up $\mathcal{M}^3$, obtaining the explicit form of metric g, curvature $\mathcal{R}$, sechsbein etc.
- Cross-check: the Euler number

$$\chi(T^6/Z_3) = 27\chi(\mathcal{M}^3) = \frac{1}{3}\int_{\mathcal{M}^3} \mathrm{tr}\left(\frac{\mathcal{R}}{2\pi i}\right)^3 = -72$$

- Finally we can explicitly define a $U(1)$ bundle $\mathcal{F}$ on the space, and embed it into the $SO(32)$ gauge group as in the previous case $F = V^I H^I \mathcal{F}$.
- The H Bianchi identity fixes the maximum amount of flux, and we get $\bar{V}^2 = 12$.
- From V^I we can compute the unbroken gauge group (commutant with $V^I H^I$), and thus, from the explicit form of $\mathcal{R}$ and $\mathcal{F}$, the chiral spectrum through the reduction of the 10d anomaly polynomial (Dirac index).

Matching the classifications:
T^6/Z_3 orbifold vs its blow-up smooth space

Orbifold models: $V^2 = 0 \bmod 6$	Blow up models: $\bar{V}^2 = 12$
$(0^{16}) \Rightarrow SO(32)$	no match
$(0^{13}, 1^2, 2) \Rightarrow SO(26) \times U(3)$	$(0^{12}, 3, 1^3) \Rightarrow SO(24) \times U(1) \times U(3)$ $(0^{13}, 2^3) \Rightarrow SO(26) \times U(3)$
$(0^{10}, 1^4, 2^2) \Rightarrow SO(20) \times U(6)$	$(0^{10}, 1^4, 2^2) \Rightarrow SO(20) \times U(4) \times U(2)$
$(0^7, 1^6, 2^3) \Rightarrow SO(14) \times U(9)$	$(0^7, 1^8, 2) \Rightarrow SO(14) \times U(8) \times U(1)$
$(0^4, 1^8, 2^4) \Rightarrow SO(8) \times U(12)$	$(0^4, 1^{12}) \Rightarrow SO(8) \times U(12)$ $\frac{1}{2}(3^4, 1^{12}) \Rightarrow U(4) \times U(12)$
$(0, 1^{10}, 2^5) \Rightarrow U(1) \times U(15)$	$\frac{1}{2}(1^{14}, 3, -5) \Rightarrow U(14) \times U(1)^2$

Fineprints & caveats: details of a case

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Orb.	$SO(26) \times U(3)$	$3(\mathbf{26}, \overline{\mathbf{3}})_{-1} + 3(\mathbf{1}, \overline{\mathbf{3}})_{2} +$ $27\,\{3(\mathbf{1}, \mathbf{3})_0 + (\mathbf{1}, \mathbf{1})_2 + (\mathbf{26}, \mathbf{1})_{-1}\}$
Sm. I	$SO(26) \times U(3)$	$3(\mathbf{26}, \overline{\mathbf{3}})_{-1} + 26 \times 3(\mathbf{1}, \mathbf{3})_{-2}$
Sm. II	$SO(24) \times U(3) \times U(1)$	$3(\mathbf{24}, \overline{\mathbf{3}})_{-1} + 6(\mathbf{1}, \overline{\mathbf{3}})_2$ $26 \times 3(\mathbf{1}, \mathbf{3})_4 + +(\mathbf{24}, \mathbf{1})_3$

- First smooth model: matching at the chiral spectrum level, vev for the singlet twisted field.
- Second smooth model: matching at the chiral spectrum level, vev for the $(\mathbf{26}, \mathbf{1})$ twisted field.
- No match for the $U(1)$ gauge charges! (But they are anyway either Higgs-broken or anomalous – that's the same ...)

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- The matching between orbifold models and smooth models with $U(1)$ flux has been shown (in some simple case).
- The analysis was performed
  - on a "semi-explicit" way, by using the topological properties of the smooth space only (valid and checked in the $K3 - T^4/Z_n$ case).
  - in an explicit way, by studying the blow-up of orbifold singularities (valid and checked in the T^{2n}/Z_n case.)

$\Rightarrow$ Extension to other geometries?

$\Rightarrow$ Beyond the matching: moduli stabilization in orbifold model building through the "smooth approach".