Supersymmetric
And Non-supersymmetric
Finite Non-Renormalizable Theories

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- In this talk I report about some results on the investigation of non-renormalizable theories
- I show that in some cases it is possible to give sense to power-counting non-renormalizable theories and work with them

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It is also an example of asymptotically safe theory

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- The theories are not conformal because they contain couplings with negative dimensionalities in units of mass

Outline of the talk

- Part I: Finiteness of quantum gravity coupled with matter in three spacetime dimensions

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- Part II: Finite chiral non-renormalizable deformations of interacting superconformal field theories in D=4
- Part III: “Quasi finite” theories in D=4, such as the Pauli deformation of Yang-Mills theory

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$$L = L_C[\varphi, \alpha] + \sum_i \kappa^i \sum_I \lambda_{iI} O_{iI}$$

$[\kappa] = -1$ α = marginal couplings

Beta functions

Dimensional counting ensures that the beta functions have the form

$$\beta_{iI} = \sum_{\{n_{jJ}{}^{iI}\}} f_{\{n_{jJ}{}^{iI}\}}(\alpha) \prod_{j \le 1} \prod_{J=1}^{N_j} \left(\lambda_{jJ}\right)^{\{n_{jJ}{}^{iI}\}}$$

where $$\sum_{j \le i} j \sum_{J=1}^{N_j} n_{jJ}{}^{iI} = i$$

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Each β depends on its own λ only linearly. Schematically,

$$\beta_\lambda = \gamma_\lambda(\alpha)\lambda + \delta_<(\alpha,\lambda)$$

where γ is the anomalous dimension of λ and $\delta_<$ is the set of terms depending on the λs with lower dimensionalities

Finiteness conditions

The *finiteness conditions* are $\beta_\lambda = 0$, which imply

$$\lambda = -\frac{\delta_<(\alpha,\lambda)}{\gamma_\lambda(\alpha)} = -\frac{\overline{\delta}_<(\alpha)}{\gamma_\lambda(\alpha)}$$

There exist solutions whenever $\gamma \neq 0$, if $\delta_< \neq 0$, otherwise when $\delta_< = 0$

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- The theory must interact!
- The conformal invariance of the renormalizable subsector ensures that the conditions can be studied algorithmically

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- The solution is non-trivial if there exists a finite irrelevant operator

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$$\Phi^n, \qquad \gamma_{\Phi^{\mathrm{n}}} = n\gamma_{\Phi}$$

- Finite operators are the stress-tensor, the gauge currents, the chiral operators of superconformal field theories

$$\gamma_{\Phi} = 0$$

$$L = L_C[\varphi, \alpha] + \kappa^{\ell} O_{\ell}(\varphi) - \sum_{i=2} \kappa^{i\ell} \frac{\overline{\delta}_{i\ell}(\alpha)}{\gamma_{i\ell}(\alpha)} O_{i\ell}(\varphi)$$

$$\gamma_{i\ell} \approx \alpha \qquad \delta_{\ell} = 0 \qquad \lambda_{2\ell} = -\frac{\delta_{2\ell}}{\gamma_{2\ell}} \approx \frac{1}{\alpha}$$

$$\delta_{i\ell} \approx \prod_{j<i} \lambda_{j\ell}^{n_j} \qquad \sum_{j<i} j n_j = i \qquad \sum_{j<i} n_j \geq 2$$

General behavior : $\lambda_{j\ell} \approx \frac{1}{\alpha^{j-1}}$

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The proof is by induction:

$\lambda_{j\ell} \approx \dfrac{1}{\alpha^{j-1}}$ for $j < n$ implies

$$\lambda_{n\ell} \approx \frac{\delta_{n\ell}}{\gamma_{n\ell}} \approx \frac{1}{\alpha} \prod_{j<i} \lambda_{j\ell}^{n_j} \approx \frac{1}{\alpha^{1+\sum_{j<i}(j-1)n_j}} = \frac{1}{\alpha^{n+1-\sum_{j<i} n_j}}$$

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$\kappa = \overline{\kappa}\alpha^{1/\ell}$ $\qquad$ $\overline{M}_P = M_P \alpha^{1/\ell}$ = "effective Planck mass"

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- The operators constructed with the curvature tensors are finite (from the C viewpoint), but not protected

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D=4:

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- So, quantum gravity coupled with matter in D=3 can be quantized as a finite theory

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- Lagrangian

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$$+\kappa\lambda_1\frac{e}{4}\left(\overline{\psi}\psi\right)^2 + \kappa\lambda_2\frac{e}{4}\left(\overline{\psi}\gamma^a\psi\right)^2 + O(\kappa^2)$$

The two-loop results give

$$\lambda_{1B} = \lambda_1 + \frac{g^4 n_f (8\lambda_1 + 9\lambda_2)}{96\pi^2 \varepsilon}, \qquad \lambda_{2\mathrm{B}} = \lambda_2 + \frac{g^4 n_f (12\lambda_1 - 8\lambda_2 - 5)}{384\pi^2 \varepsilon}$$

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- Deform it with the chiral non-renormalizable (finite, protected) operator

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- Determine the “renormalization queue” of the non-renormalizable perturbation solving algorithmically the finiteness equations

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- Finite theories: all couplings have Z=1, but there can be dimensionful parameters

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Example: N=4 SYMT + N=0 mass

Quasi finite non-renormalizable theories

First λ : $\beta_{\lambda_1} = \lambda_1 \gamma_1(\alpha)$ $\lambda_1 \approx \mu^{\gamma_1}$ In general, $\gamma_1 \neq 0$

Other λs : take dimensionless ratios $r_i = \dfrac{\lambda_i}{\lambda_1^{\,n_i}}, \quad n_i = \dfrac{[\lambda_i]}{[\lambda_1]}$

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Example: Pauli deformation of non-Abelian Yang-Mills theories

$$\mathcal{L} = \frac{\mu^{-\varepsilon}}{4g^2 Z_g^2} (\mathcal{F}^a_{\mu\nu})^2 + \overline{\Psi}^I_i \not{D}_{ij} \Psi^I_j,$$

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$$\frac{\beta_g}{g} = -\frac{\Delta}{3}\frac{g^2 N_c}{16\pi^2} + \frac{25}{2}\left(\frac{g^2 N_c}{16\pi^2}\right)^2 + \sum_{n=3}^{\infty} c_n \left(\frac{g^2 N_c}{16\pi^2}\right)^n$$

$$\frac{g_*^2 N_c}{16\pi^2} = \frac{2}{75}\Delta + \mathcal{O}(\Delta^2) \qquad \Delta \equiv 11 - 2N_f/N_c \ll 1$$

Lowest level: Pauli term

$$\mathcal{L}_{\rm Pauli} = \kappa\lambda Z_\lambda\, \mathcal{F}^a_{\mu\nu}\; \overline{\Psi}^I_i T^a_{ij} \sigma_{\mu\nu} \Psi^I_j,$$

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$$\beta_\lambda = \frac{g^2\lambda(N_c^2-5)}{16\pi^2 N_c} \sim \frac{2}{75}\lambda\Delta$$

$$\lambda(\Lambda) = \lambda(\mu)\left(\frac{\Lambda}{\mu}\right)^{2\Delta/75}.$$

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Second level: Fcube plus four-fermion terms

$$\mathcal{L}_{F^3} = \frac{\kappa^2\mu^{-\varepsilon}}{6!}\zeta Z_\zeta\; f^{abc}\mathcal{F}^a_{\mu\nu}\mathcal{F}^b_{\nu\rho}\mathcal{F}^c_{\rho\mu}$$

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$$u \equiv \frac{\zeta}{\lambda^2}$$

$$\zeta = \frac{11 \lambda^2}{5 g_*^2} = \frac{165}{2} \frac{1}{\Delta} \left(\frac{\lambda^2 N_c}{16\pi^2} \right)$$

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$$\mathrm{S} = (\overline{\Psi}^I_i\Psi^I_i)^2,\quad \mathrm{P} = (\overline{\Psi}^I_i\gamma_5\Psi^I_i)^2,\quad \mathrm{V} = (\overline{\Psi}^I_i\gamma_\mu\Psi^I_i)^2,\quad \mathrm{A} = (\overline{\Psi}^I_i\gamma_5\gamma_\mu\Psi^I_i)^2,$$

$$\mathrm{T} = (\overline{\Psi}^I_i\sigma_{\mu\nu}\Psi^I_i)^2,\quad \mathrm{S}' = (\overline{\Psi}^I_i\Psi^I_j)(\overline{\Psi}^I_j\Psi^I_i),\quad \mathrm{P}' = (\overline{\Psi}^I_i\gamma_5\Psi^I_j)(\overline{\Psi}^I_j\gamma_5\Psi^I_i),$$

$$\mathrm{V}' = (\overline{\Psi}^I_i\gamma_\mu\Psi^I_i)^2,\quad \mathrm{A}' = (\overline{\Psi}^I_i\gamma_5\gamma_\mu\Psi^I_i)^2,\quad \mathrm{T}' = (\overline{\Psi}^I_i\sigma_{\mu\nu}\Psi^I_j)(\overline{\Psi}^I_j\sigma_{\mu\nu}\Psi^I_i).$$

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- What is necessary for the construction to work? One finite operator with dimensionality greater than four
- When does the construction not work? When there exist infinitely many finite non-protected operators