Optics and Spectroscopy, Vol. 95, No. 4, 2003, pp. 631–637. Translated from Optika i Spektroskopiya, Vol. 95, No. 4, 2003, pp. 675–682.

PHYSICAL AND QUANTUM OPTICS

On Accounting for the Effect of Particles of a Condensed Dispersed Phase on Radiant Energy Transfer in Gaseous Media

N. N. Bezuglov*, A. K. Kazanskiĭ*, A. N. Klyucharev*, K. Michulis, F. Fuzo***, and M. Allegrini*****

** Research Institute of Physics (Petrodvorets Branch), St. Petersburg State University, Peterhof, 198504 Russia*

*** Institute of Atomic Physics and Spectroscopy, University of Latvia, LV-1586 Riga, Latvia*

**** National Institute for Nuclear Physics, Fermi Department of Physics, University of Pisa, I-56127 Pisa, Italy*

Received January 8, 2003

Abstract—Questions concerning the formation of the optical properties of dense gaseous and plasma media in relation to the specific features of radiant energy transfer are considered. The integral equations describing the radiation trapping are investigated as a new class of generalized wave equations of Schrödinger type. Starting from the methods of quantum mechanics, original analytical and numerical approaches are suggested for solving problems of the radiative kinetics of both spatially homogeneous and inhomogeneous absorbing media containing dispersed particles. In terms of the quasi-classical approximation, two classes of reference problems for determination of phase factors are formulated. Solutions for a number of model problems are presented that demonstrate the efficiency of the methods developed. © *2003 MAIK "Nauka/Interperiodica"*.

INTRODUCTION

As is well known [1, 2], the trapping of resonance radiation radically affects the formation of luminescence of optically dense gaseous and plasma media. In this context, a great number of studies are devoted to different aspects of radiant energy transfer both in planet atmospheres [3] and under the conditions of modern spectroscopic experiments [4, 5]. Studies of the last decade [6, 7] revealed also the importance of processes of multiple reemission of photons in magneto-optical traps, where radiation trapping in the range of operating frequencies of cooling lasers prevents obtaining high concentrations and low temperatures of the atoms cooled [8].

The presence of dispersed particles of a substance in either solid or liquid state in a plasma may lead to significant changes in the electrokinetic properties of the plasma medium. In the general case, the dynamics of such a multiphase system should also consider the processes of radiant energy exchange between the (carrier) medium and the macroparticles and microparticles of the substance. The interest in investigations of low-temperature plasmas containing a condensed dispersed phase (CDP) is caused by the use of such media in propulsion and energy systems operating on solid and liquid fuels and in modern high technologies.

Among the numerous known methods of diagnostics of CDP-containing plasmas, spectroscopic methods are the simplest and most universal. On the basis of experimental data on the intensities and shapes of spectral lines and the optical thickness of the absorbing layer in the medium studied, these methods yield reliable information about the parameters of the object under investigation. In most cases of practical importance, the gas studied is nontransparent for radiation emerging from it. Such media, in which the intrinsic absorption (self-absorption) of spectral lines plays an important role, are called optically dense. The influence of CDP particles on the optical properties of a plasma, including their effect on the characteristics of radiation transfer under conditions when a flow of ionized gas with evaporating drops of an alkali metal exists, was considered in [9]. The results obtained made it possible to develop a technique of optical diagnostics of a plasma with a CDP in the form of local formations having the structure of a metal nucleus surrounded by a vapor shell. Such a simulation of a plasma with a CDP is also of interest from the practical point of view, since the phenomenon of nonequilibrium ionization was observed previously under the same conditions. The model of radiation transfer in a strongly inhomogeneous plasma with a CDP in the form of an evaporating liquid-metal drop, suggested in [9], was confirmed experimentally for a number of informative parameters.

We should note that media in which mass transfer phenomena play an important role are convenient objects for observation of new spectroscopic effects. The specific features of the shapes of spectral lines in the system of a drop evaporating in a vacuum may serve as an example. In the latter case, the drop, for example, of an alkali metal, is the source of an expanding vapor shell composed of alkali atoms. The concentration of atoms is highest at the drop surface and rapidly decreases with increasing distance from the surface. The characteristics of absorption and emission of light within the shell are governed by both the spatial and

0030-400X/03/9504-0631$24.00 © 2003 MAIK "Nauka/Interperiodica"

velocity distributions of atoms. In this case, the emission line may become narrower in comparison with the absorption line if, within the lifetime of the excited state, an atom that absorbed a photon near the surface of the liquid-metal nucleus passes to a region where the concentration of normal atoms is significantly lower than its maximum value near the surface. As a result, the case of an "optical condenser" may be realized in the frequency space (the system absorbs light in a wide spectral range and reemits light in a narrow range of frequencies [10]).

Thus, problems of fundamental and applied physics for the typical conditions of optically dense media exist whose solution requires rigorously accounting for the processes of radiation trapping (transfer). We present below the results of our studies based on the application of new approaches to solving problems of the radiation kinetics of gaseous media with both uniform and nonuniform spatial distributions of the absorption and emission coefficients of resonance radiation. It should be noted that accounting for the spatial inhomogeneity of spectral characteristics in calculations of the distribution of excited atoms is the most complex problem and, as will be shown below, should be taken into account in terms of the reference problem method.

INTEGRAL EQUATION FOR RADIATION TRAPPING

The simplest equation simulating radiation trapping in the approximation of a two-level scheme of an atom is the Biberman–Holstein equation [2, 11]

$$\frac{d}{dt}n^*(\mathbf{r}, t) = -[A_{21} + W(\mathbf{r})]n^*(\mathbf{r}, t) + A_{21}\int_\Omega d^3\tilde{r}G(\mathbf{r}, \tilde{\mathbf{r}})n^*(\tilde{\mathbf{r}}, t) + \alpha^*(\mathbf{r}, t), \quad (1)$$

$$G(\mathbf{r}, \tilde{\mathbf{r}}) = \frac{1}{4\pi\rho^2}\int_{-\infty}^{\infty} k(\nu, \mathbf{r})\varphi(\nu, \tilde{\mathbf{r}})\exp[-\chi(\nu, \mathbf{r}, \tilde{\mathbf{r}})]d\nu, \quad \rho = |\mathbf{r} - \tilde{\mathbf{r}}|. \quad (2)$$

Here, A_{21} and W are, respectively, the probabilities of the radiative decay and quenching of the population $n^*(\mathbf{r}, t)$ of the resonance level at the spatial point $\mathbf{r}$ per unit time and α^* (the so-called function of primary sources) sets the excitation rate of atoms enclosed in a cell of volume Ω. The nucleus G of the integral term is governed by the probability of absorption of a photon in the vicinity of the point of observation $\mathbf{r}$ under the condition of photon emission in the vicinity of the point $\tilde{\mathbf{r}}$; G contains the profiles of the spectral coefficients of absorption $k(\nu, \mathbf{r})$ and emission $\varphi(\nu, \tilde{\mathbf{r}})$ of photons, normalized by the condition $\int\varphi(\nu, \mathbf{r})d\nu = 1$. The exponential factor yields the reduction in intensity of light of frequency ν when it passes the segment between the points $\tilde{\mathbf{r}}$ and $\mathbf{r}$ with the optical thickness χ

$$\chi(\nu, \mathbf{r}, \tilde{\mathbf{r}}) = \int_r^{\tilde{r}} k(\nu, \mathbf{r}_l)dl. \quad (3)$$

On the assumption of complete frequency redistribution [3], the profiles $\kappa(\nu, \mathbf{r})$ and $\varphi(\nu, \mathbf{r})$ are proportional to each other:

$$\varphi(\nu, \mathbf{r}) = C_n(\mathbf{r})\kappa(\nu, \mathbf{r}), \quad C_n(\mathbf{r}) = 1/\int_{-\infty}^{\infty}\kappa(\nu, \mathbf{r})d\nu. \quad (4)$$

The functional representations of the spectral line shapes of the most significant types have the forms [3, 4]

$$\kappa(\nu) = \kappa_0\exp(-\nu_c^2), \quad \nu_c = \frac{\nu - \nu_0}{\gamma_D} \quad \kappa_0 = \kappa_0^{(D)};$$
$$\kappa(\nu) = \kappa_0\frac{1}{1 + \nu_c^2}, \quad \nu_c = \frac{\nu - \nu_0}{\gamma_L} \quad \kappa_0 = \kappa_0^{(L)};$$
$$\kappa(\nu) = \kappa_0\frac{a}{\pi}\int_{-\infty}^{\infty}\frac{\exp(-y^2)}{a^2 + (\nu_c - y)^2}dy, \quad \nu_c = \frac{\nu - \nu_0}{\gamma_D}, \quad a = \frac{\gamma_L}{\gamma_D} \quad (5)$$

for the Doppler, Lorentz, and mixed (Voigt) mechanisms of line broadening, respectively. Here, γ_L and $\gamma_D = c^{-1}\nu_0\sqrt{2kT/m}$ are the Lorentz and Doppler line widths, respectively; c is the speed of light; k is the Boltzmann constant; m is the molecular mass; and $\kappa_0^{(D)}$ and $\kappa_0^{(L)}$ are the absorption coefficients at the line center ν_0 for the Doppler and Lorentz lines, respectively. In the case of a mixed shape, the absorption coefficient coincides with that for the Doppler line: $\kappa_0 = \kappa_0^{(D)}$.

It should be noted that solving integrodifferential equations of type (1) involved serious difficulties even in the case of a two-level model of an atom and homogeneous media in cells of simplest form. Physically, these difficulties are caused by the possibility of light energy exchange between distant atoms due to the photon deexcitation in the line wings, which are transparent for gaseous media. The formal consequence of the existence of the long-range part of the spectrum is the divergence of the photon mean free path, which makes invalid the approximation of the integral trapping equations by local diffusion equations of Fokker–Planck type. For this reason, many conventional numerical schemes become inadequate as applied to the general class of integrodifferential transport equations [12]. More or less universal approaches [4, 13], based on the numerical simulation of processes in terms of the Monte Carlo method, require efficient computers and much computing time. At the same time, the algorithms with acceptable computational speed, developed for astrophysical applications, are rather specific [3, 14] and applicable only to solving a narrow range of problems (one-dimensional geometries of the plane-atmosphere type). The exceptions to this rule are situations when the radiation trapping occurs with conservation of the frequency of reemitted photons [15] or when the processes of transfer and scattering of light energy occur in turbid media with conservation of the photon mean free path within the line [1, 16]. In this case, the kinetic equations of the Biberman–Holstein type are reduced to one or another modification of the diffusion equations (the Milne equations [1]), which accounts for the fact that the corresponding theories are highly advanced.

METHOD OF GENERALIZED WAVE EQUATIONS FOR PROBLEMS OF THE RADIATION KINETICS OF GASEOUS MEDIA

In view of the above considerations, it is of interest from both the practical and the scientific points of view to develop radically new approaches (numerical and analytical), which would, on the one hand, lead to simple calculations and, on the other hand, be sufficiently universal. In this study, we describe such a method [17–20], which will be referred to as the generalized-wave-equation approach (GWEA). In terms of the GWEA, the integral equations are considered as a new class of wave equations for some classical three-dimensional Hamiltonian system (quasiparticles). Such an interpretation makes it possible, primarily, to develop a modified quasi-classical approximation (a method of geometrical quantization) and obtain an analytical description of the trapping effects in regions with separable variables (a plane layer; a sphere; a finite cylinder; and various parallelepipeds, prisms, and ellipses). Recently, a new numerical method for solving trapping equations based on fast algorithms (the split propagation technique) was suggested [21, 22], aimed at accurate investigation of nonstationary problems of radiation transfer in convex regions.

The concept of the GWEA can be followed most easily through the example of solving a spectral problem for the Biberman–Holstein equation (1), which appears in the case of expansion of the level population in a Fourier series [4, 11]:

$$n^*(\mathbf{r}, t) = \Sigma_j \alpha_j \psi_j(\mathbf{r}) \exp(-A_{21} t/g_j). \qquad (6)$$

Here, the quantities ψ_j are set by the normalized eigenfunctions (modes) of the trapping equation; $1/g_j = \lambda_j$ are the corresponding eigenvalues; and the expansion coefficients α_j are governed by the initial spatial distribution of the excited states. The parameters g_j are referred to as the trapping factors; their values are related to the average number of reemission events in the jth mode. The radiation modes, i.e., the complete set of the functions ψ_j, are found by solving the eigenvalue problem (the so-called spectral problem) for trapping equation (1) [4, 11]:

$$A_{21}\lambda_j\psi_j(\mathbf{r}) = A(\hat{I} - \hat{G})\psi_j(\mathbf{r}) + W(\mathbf{r})\psi_j(\mathbf{r}), \qquad (7)$$

where the symbols $\hat{G}$ and $\hat{I}$ correspond to the integral trapping operator and the unit operator, respectively. The subscript j enumerates the modes. The function $\psi_j(\mathbf{r})$ sets the spatial profile of the jth mode, whereas the quantity λ_j controls the effective constant of its radiative decay. The radiation -kinetics problem can be considered solved if one or another algorithm for calculation of the complete set (spectrum) of effective radiation constants λ_j and corresponding modes $\psi_j(\mathbf{r})$ in the volume Ω is found.

In [17, 18], an analogy between the general form of Eqs. (1) and (7) and some class of differential equations was noted. Thus, it is suggested that trapping equation (7) be considered as a variant of a steady-state wave equation for a three-dimensional classical system (a quasiparticle), which is set by a Hamiltonian associated with (1) and (7),

$$H(\mathbf{r}, \mathbf{p}) = A_{21}\tilde{V}(\mathbf{r}, \mathbf{p}) + W(\mathbf{r}), \qquad (8)$$

$$\tilde{V}(\mathbf{r}, \mathbf{p}) = 1 - \frac{1}{|\mathbf{p}|}\int_{-\infty}^{\infty} \varphi(\nu, \mathbf{r}) k(\nu, \mathbf{r}) \arctan\left(\frac{|\mathbf{p}|}{k(\nu, \mathbf{r})}\right) d\nu. \qquad (9)$$

Note that, in the absence of quenching, this fact is obvious for the case of an infinite homogeneous space $\Omega_3^{(\infty)}$, since the nucleus G in (2) depends on the difference $\rho = |\mathbf{r} - \tilde{\mathbf{r}}|$ of its arguments. In this case, integral operator (1) is of convolution type and its eigenfunctions are the plane waves $\exp(i\mathbf{r}\mathbf{p})$. The corresponding eigenvalues $A_{21}\lambda_p = A_{21}\tilde{V}(\mathbf{p})$ are the amplitudes of the Fourier transform of the nucleus G, which have the same form as expression (9) [3]. This circumstance allows one to consider the wave vector $\mathbf{p}$ as the momentum and the eigenvalue $A_{21}\lambda_p$ as the kinetic energy in the system of units with Planck's constant $\hbar = 1$. In another formulation, this fact means that the action of the operator $(\hat{I} - \hat{G})$ in the momentum space reduces to multiplication of the functions by the factor $\tilde{V}(\mathbf{p})$. Taking into account that, in the coordinate space, the action of the momentum operator reduces to differentiation, $\mathbf{p} = -i\partial/\partial\mathbf{r}$, spectral problem (7) can be rewritten in the equivalent form

$$H(\mathbf{r}, -i\nabla)\psi_j(\mathbf{r}) = \lambda_j A_{21}\psi_j(\mathbf{r}) \qquad (10)$$

with the Hamiltonian H (8). The similarity of this equation with the Schrödinger equation is obvious. When one goes to generalized wave equation (10), two new aspects of the problem arise. First, it is possible to con-

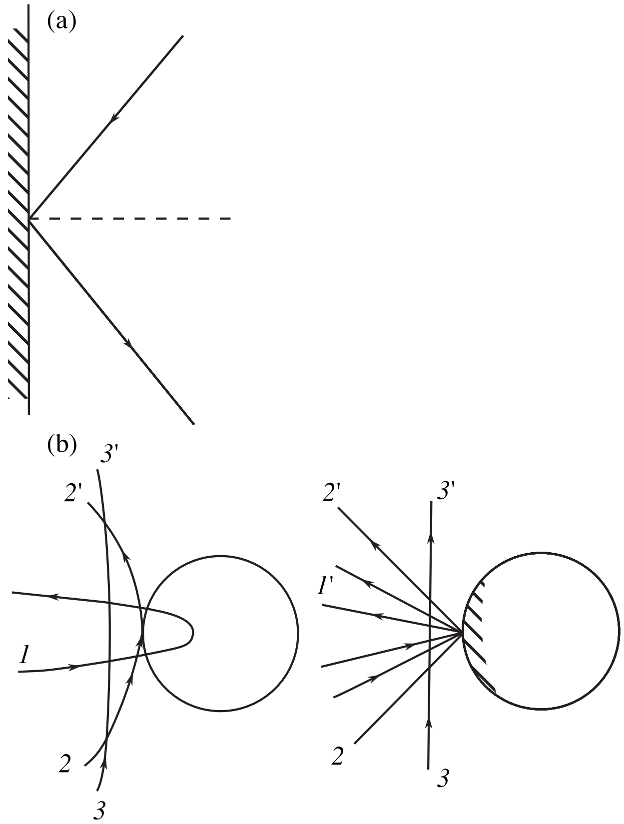

Fig. 1. Path of motion of a quasiparticle in a gaseous medium with CDP particles: (a) elastic reflection of the quasiparticle from a planar potential surface, corresponding to a cell wall, and (b) actual paths of a quasiparticle in the vicinity of CDP particles and the corresponding billiard paths.

sider the quenching probability $W(\mathbf{r})$ as the potential energy in the Hamiltonian equations of motion

$$\frac{d}{dt}\mathbf{r} = \frac{\partial}{\partial \mathbf{p}}H(\mathbf{r}, \mathbf{p}), \quad \frac{d}{dt}\mathbf{p} = -\frac{\partial}{\partial \mathbf{r}}H(\mathbf{r}, \mathbf{p}) \qquad (11)$$

for a quasiparticle. The rigorous justification of the latter statement was performed in [23] on the basis of the method of continuum Feynman integrals, where, in particular, it was shown that the canonical scheme of quantization of Hamiltonian (8) in an unlimited homogeneous space $\Omega_3^{(\infty)}$ leads to the identical spectral problem (7) for trapping equation (1). The second important circumstance arises in the case of media characterized by substantial spatial inhomogeneity and is related to the problem of ordering the operators $\hat{\mathbf{p}} = -i\partial/\partial\mathbf{r}$ and $\hat{\mathbf{r}}$ (which do not commute with each other) in Eq. (10) (more exactly, in the function $\tilde{V}(\mathbf{r}, \mathbf{p})$ (9)). The correct ordering of operators can be carried out on the basis of two requirements: (i) spectral problem (10) should correspond to a self-adjoint operator [24] and (ii) Eq. (10) should be consistent with the homogeneous trapping equation in terms of the so-called variable reduced optical thicknesses [25–27]. However, we should note that, in terms of quasi-classical methods of solving problems of type (10), Hamiltonian (8) and paths (11) are determined unambiguously [24].

RADIATION KINETICS AS A VARIANT OF THE QUANTUM BILLIARD PROBLEM

In [17], an approach is described that allows one to extend the concept of a quasiparticle to the case of finite gas-filled volumes. In order to do this, it is sufficient to assume the quenching probability W to be equal to infinity beyond the volume Ω and to extend the limits of integration in (1) to the entire space $\Omega_3^{(\infty)}$, since $n^*(\mathbf{r}) = 0$ in the regions where $W(\mathbf{r}) = \infty$. Now let us return to the interpretation of W as the potential energy. An infinite drop of the potential W at the boundary $\partial\Omega$ of the cell volume Ω means the confinement of a quasiparticle in the potential box Ω. At the same time, the appearance of an impermeable potential wall $\partial\Omega$ leads to elastic reflection of the paths (11) from the cell boundary (Fig. 1a). Thus, an important and rather unexpected consequence of the wave–particle dualism (with the wave and particle described by Eqs. (7) and (11), respectively) is the complementarity principle with respect to the transparency of the cell walls: the fact of free escape of photons from the gas volume Ω (the absence of light reflection from the walls) means the appearance of an infinite, impermeable for the particle (11), potential barrier at the boundary $\partial\Omega$ of the absorbing medium.

In the case of a low-temperature plasma with a CDP, in the vicinity of CDP particles, abrupt changes both in the concentration and in the temperature of absorbing atoms may be observed (the typical situation for the alkali aerosol plasma mentioned in the Introduction). With regard to the quasiparticle, this circumstance should lead to strong refraction effects for its path characteristics (11), which is shown schematically in Fig. 1b as scattering (reflection) of a quasiparticle from the surface of a CDP particle. The path of motion for the Hamiltonian $H(\mathbf{r}, \mathbf{p})$ (8) consists of segments of lines restricted by the cell walls and the surfaces of CDP particles. Thus, the problem of determination of the effective radiation constants of the trapping equation reduces to determination of the quantum energy levels $A_{21}\lambda_j$ for a point quasiparticle with paths (11), placed in the three-dimensional "billiard" Ω with an elastically reflecting surface $\partial\Omega$ and scattering centers in the form of randomly distributed spheres (CDP particles) (Fig. 1b).

Obviously, the kinetic energy $\tilde{V}$ plays an important role in this case. The specific features of the behavior of this quantity (as a function of the momentum $\mathbf{p}$) set the dispersion characteristics of a quasiparticle and, in many respects, govern the specificity of the GWEA for radiation transfer problems. Different analytical representations for $\tilde{V}$ [18] show, in particular, that the quasiparticle behavior is characterized by a complex dispersion law. In the Fourier space, small magnitudes of the momentum **p** correspond to an extended region with the characteristic linear size L ($|\mathbf{p}| \sim 1/L$). Therefore, large optical thicknesses τ correspond to large values of the parameter $\kappa_0/|\mathbf{p}| \sim \tau$, which enters the argument of the function $\tilde{V}$ (9). The properties of the trapping factors for optically dense media are governed by the behavior of the absorption coefficient $\kappa(\nu)$ in the line wings [3]. The energy exchange between atoms due to radiation transfer at the frequencies of the spectral lines of atoms belongs to the class of long-range interactions. The formal consequence of the divergence of the mean free path of photons is the violation of the smoothness of the curve of the kinetic energy $\tilde{V}$ at small momenta **p**. This statement can be easily illustrated for the cases of Lorentz and Doppler shapes of lines [3, 18]:

$$\tilde{V}_L(\mathbf{p}) \simeq \frac{\sqrt{2}}{3}\sqrt{|\mathbf{p}|/\kappa_0^{(L)}},$$

$$\tilde{V}_D(\mathbf{p}) \simeq \frac{\sqrt{\pi}}{4}\frac{|\mathbf{p}|}{\kappa_0^{(D)}}\frac{1}{\sqrt{\ln(\kappa_0^{(D)}/|\mathbf{p}|)}}, \quad |\mathbf{p}|/\kappa_0 \longrightarrow 0.$$

As can be seen, the velocity $\mathbf{v} = \partial/\partial\mathbf{p}\,\tilde{V}$ of a quasiparticle (the so-called group velocity for the wave packets) tends to infinity in the vicinity of the values of momenta $|\mathbf{p}| = 0$.

NONCONVENTIONAL METHODS FOR SOLVING THE RADIATION TRANSFER EQUATION

Another important consequence of the above consideration is that trapping equation (1) can be written in the form of the nonstationary equation

$$\frac{d}{dt}n^*(\mathbf{r}, t) = -H(\mathbf{r}, -i\nabla)n^*(\mathbf{r}, t) + \alpha^*(\mathbf{r}, t) \quad (12)$$

in order to determine the evolution of a generalized quantum-mechanical system (a quasiparticle). The above reformulation of the radiation transfer problem reveals the radically new possibility of constructing solutions to problems of radiation kinetics with the use of recently developed methods of computational physics.

The numerical calculation of the dynamics of excited states can be performed on the basis of the split propagation technique (SPT), which has been well developed recently [28]. The universal algorithm and its realization in a specific program for solving Eq. (12) are described in [21]. The advantages of the numerical scheme [21] are its efficiency in study of arbitrary convex volumes with absorbing media and the possibility of analyzing spectral problem (10). As an example,

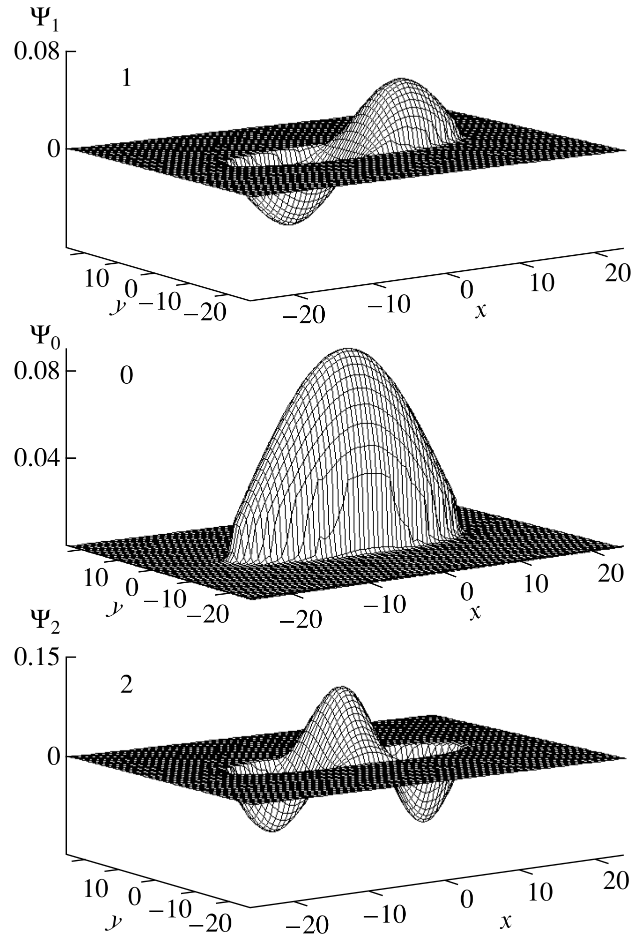

Fig. 2. Spatial profiles of first three modes $\psi_j(x, y)$ ($j = 0, 1, 2$) of the trapping equation for the case of a gas-filled cell in the form of an elliptical infinitely high cylinder. The ratio of the ellipse semiaxes $R_>/R_<$ in the cylinder base is 2. The line has Lorentz shape, and the optical thickness with respect to the semimajor axis is $\tau = \tau = \kappa_0^{(L)}R_> = 10$.

Fig. 2 shows the results of calculations of the spatial behavior of the first three modes of the trapping equation for homogeneous gaseous media of elliptical shape. It should be noted that the SPT makes it possible to analyze the radiation transfer problems stated more generally [22] under the condition of partial frequency redistribution. The further development of the method, as applied to systems of cold atoms, when the absorption-line shape is governed mainly by the natural linewidth and the mechanism of diffusion migration at the radiation frequencies is realized (which reduces the redistribution problem to an equation of Fokker–Planck type [29]), will be published elsewhere.

In the analytical methods for solving Eqs. (10) and (12), the quasi-classical approach to the problem of quantization of the quasiparticle energy is used [18, 19]. Here, in view of a number of specific features inherent in radiation kinetics problems, development

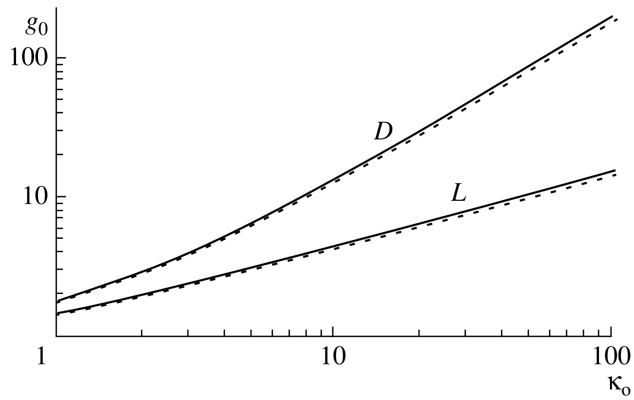

Fig. 3. Trapping factors g_0 for the fundamental mode in the case of an oblong ellipsoid of revolution with semiaxes $R_\perp = 1$ cm and $R_z = 2$ cm. The values of g_0 obtained by geometric quantization (the solid lines) are shown as functions of the absorption coefficient k_0 [cm^{-1}] at the line center. The dashed lines show the results of numerical calculations by the Monte Carlo method. The lines have Doppler and Lorentz shapes (*D* and *L*, respectively).

and refinement of special techniques is required [30]. For example, it becomes necessary to obtain the solutions for a number of reference equations describing the radiation trapping, with subsequent correction of the phase factors on the basis of these solutions. The point is that the quasi-classical formulas for the wave functions (coinciding with the spectral modes of the trapping equations) are invalid in the vicinity of discontinuity surfaces and points of contact of different components of gaseous media. The same also holds true for description of the mode behavior in the vicinity of the cell boundary, where a potential barrier is introduced in terms of the GWEA. On the whole, the two main classes of model equations of the transfer theory should be analyzed and the abrupt changes in phases of the wave packets should be determined after they pass through (or reflect from) structures with high spatial inhomogeneity. Thus, the following problems remain to be solved: (i) determination of the degree of excitation of a medium in the spectral modes near a plane surface under the conditions of partial reflection of light from it (in terms of the quasi-classical approach, such a problem corresponds to reflection of a quasiparticle from a planar potential wall (Fig. 1a)); (ii) analysis of the modes of the trapping equation in the case of a unlimited volume, in which a sphere of finite radius with partially reflecting walls has been placed (a CDP particle). The solution to the latter problem reduces to analysis of scattering of a quasiparticle from a spherical surface.

The systematic investigation of the problems arising here was started in [18, 19, 30] in terms of the quasi-classical (short-wavelength) approximation, which was well developed in quantum mechanics and optics. The Bohr–Sommerfeld quantization rules [31] for systems with separable variables and their subsequent modification—the Einstein–Brillouin–Keller quantization method—for the case of weakly nonintegrable systems [32, 33] yield an efficient algorithm for solving the spectral problem, i.e., for analytical calculation of all the effective radiation damping constants $A_{21}\lambda_j$ (7) and the corresponding modes. Figure 3 shows the results of calculations (solid lines) of the g_0 factors ($=1/\lambda_0$) for the fundamental mode obtained by geometrical quantization [19] in the case of extended ellipsoidal cells filled with a homogeneous absorbing medium. Comparison with the numerical calculations by the Monte Carlo method (dashed curves) indicates the high accuracy (better than 5%) of geometrical quantization and, correspondingly, the promise of analytical approaches to solving radiation kinetics problems.

CONCLUSIONS

In optically dense gaseous media, the transfer of light energy within the resonance emission spectrum plays an important role in the processes of population of the resonance levels of atoms. In turn, the dynamics of the behavior of resonance states under the conditions of a steady-state gas-discharge plasma, of the afterglow stage, and of a photoplasma affects in many respects the kinetics of both the neutral and electron components of the plasma. Therefore, correctly accounting for the radiation trapping becomes an integral component of any approach aimed at constructing a closed theory of radiation kinetics for optically dense plasma media. In this study, we discussed the nonconventional method of generalized wave equations for solving integral equations, which, in our opinion, yields a basis for development of fast and efficient algorithms of calculation (analytical and numerical) of populations of the resonance levels of atoms. We should note the universality of the generalized transport equations (10) and (12), in terms of which simulation of the formation of the optical properties of absorbing media is possible for a wide range of conditions of modern experiments: from ultracold atoms in magneto-optical traps [5, 20] to plasma media in MHD generators [9]. The most complex problem is the investigation of the transfer processes in spatially inhomogeneous media. This problem can be solved on the basis of the methods and concepts described here.

ACKNOWLEDGMENTS

This study was supported by the Russian Foundation for Basic Research (project no. 02-02-16586) and by INTAS (grant no. 01-0155)

We are also grateful to the NATO Research Fund for support in terms of Collaborative Linkage Grant PST.CLG.979120.

REFERENCES

1. A. C. G. Mitchell and M. W. Zemansky, *Resonance Radiation and Excited Atoms* (Cambridge Univ. Press, Cambridge, 1961; ONTI, Moscow, 1937).
2. L. M. Biberman, V. S. Vorob'ev, and I. T. Yakubov, *Kinetics of Nonequilibrium Low-Temperature Plasmas* (Nauka, Moscow, 1982; Consultants Bureau, New York, 1987).
3. V. V. Ivanov, *Radiation Transfer and Celestial Bodies Spectra* (Nauka, Moscow, 1969).
4. A. F. Molisch and B. P. Oehry, *Radiation Trapping in Atomic Vapours* (Oxford Univ. Press, Oxford, 1998).
5. A. Fioretti, A. F. Molish, J. H. Mueller, *et al.*, Opt. Commun. **149**, 415 (1998).
6. T. Walker, D. Sesko, and C. Wieman, Phys. Rev. Lett. **64**, 408 (1990).
7. D. W. Sesko, T. G. Walker, and C. E. Wieman, J. Opt. Soc. Am. B **8**, 946 (1991).
8. K. Ellinger and J. Cooper, Phys. Rev. A **55**, 4351 (1997).
9. R. I. Asadullina, N. N. Bezuglov, A. N. Klyucharev, and V. Yu. Sepman, Opt. Spektrosk. **76**, 572 (1994) [Opt. Spectrosc. **76**, 509 (1994)].
10. R. N. Asadullina, I. N. Bebelin, N. N. Bezuglov, *et al.*, USSR Inventor's Certificate No. 283820 (3 October 1988).
11. T. Holstein, Phys. Rev. **72**, 1212 (1947).
12. *Theoretical and Applied Problems of Light Scattering*, Ed. by B. I. Stepanov and A. P. Ivanov (Nauka i Tekhnika, Minsk, 1971).
13. *Numerical Solution of Problems of Atmospheric Optics*, Ed. by Maslennikov and T. A. Sushkevich (Inst. Prikl. Mekh. Akad. Nauk SSSR, Moscow, 1984).
14. V. V. Ivanov, Astron. Astrophys. **286**, 328 (1994).
15. A. E. Bulyshev, N. E. Preobrazhenskiĭ, and A. E. Suvorov, Usp. Fiz. Nauk **156**, 153 (1988) [Sov. Phys. Usp. **31**, 865 (1988)].
16. I. N. Minin, *Theory of Radiation Transfer in Planetary Atmospheres* (Nauka, Moscow, 1988).
17. R. I. Asadullina, N. N. Bezuglov, and E. N. Borisov, Opt. Spektrosk. **67**, 360 (1989) [Opt. Spectrosc. **67**, 208 (1989)].
18. N. N. Bezuglov, A. F. Molisch, A. N. Klucharev, *et al.*, Phys. Rev. A **57**, 2612 (1998).
19. N. N. Bezuglov, A. F. Molisch, A. N. Klucharev, *et al.*, Phys. Rev. A **59**, 4340 (1999).
20. N. N. Bezuglov, A. K. Kazansky, F. Fuso, and M. Allegrini, Phys. Rev. A **63**, 042703 (2001).
21. A. K. Kazansky and N. N. Bezuglov, J. Phys. B **33**, 99 (2000).
22. A. K. Kazansky, N. N. Bezuglov, A. F. Molisch, *et al.*, Phys. Rev. A **64**, 022719 (2001).
23. N. N. Bezuglov and B. V. Taratin, Opt. Spektrosk. **84**, 893 (1998) [Opt. Spectrosc. **84**, 807 (1998)].
24. M. V. Fedoryuk, Itogi Nauki Tekh., Ser.: Sovrem. Probl. Mat. **34**, 6 (1988).
25. A. M. Dykhne and A. P. Napartovich, Preprint No. 2026, IAÉ (Inst. of Atomic Energy, Moscow, 1970).
26. N. N. Bezuglov, A. N. Klyucharev, and T. Stasevich, Opt. Spektrosk. **77**, 342 (1994) [Opt. Spectrosc. **77**, 304 (1994)].
27. N. N. Bezuglov, A. N. Klucharev, A. F. Molisch, *et al.*, Phys. Rev. E **55**, 3333 (1997).
28. M. D. Feit and J. A. Fleck, J. Chem. Phys. **80**, 2578 (1984).
29. A. E. Suvorov, Zh. Éksp. Teor. Fiz. **92**, 444 (1987) [Sov. Phys. JETP **65**, 251 (1987)].
30. N. N. Bezuglov and B. V. Taratin, Opt. Spektrosk. **86**, 360 (1999) [Opt. Spectrosc. **86**, 310 (1999)].
31. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics*, Vol. 3: *Quantum Mechanics: Non-Relativistic Theory*, 2nd ed. (Nauka, Moscow, 1963; Pergamon, New York, 1977).
32. V. I. Arnol'd, *Mathematical Methods of Classical Mechanics* (Nauka, Moscow, 1974; Springer, New York, 1989).
33. I. C. Percival, Adv. Chem. Phys. **36**, 1 (1977).

Translated by Yu. Sin'kov