Particules Élémentaires, Gravitation et Cosmologie
Année 2006-2007
String Theory: basic concepts and applications

Lecture 3: 27 February 2007

Strings and Black Holes

Outline

String vs Black-Hole entropy

h = c = numerical factors =1

M_s , l_s = string mass, length scales

Tree-level string entropy

Counting states (FV, BM ('69), HW ('70))

$$S_{st} = \frac{M}{M_s} = \frac{L}{l_s}$$

= No. of string bits in the total string length

NB: no coupling, no G appears!

Black-Hole entropy (D=4)

$$S_{BH} = MR_S = \left(\frac{R_S}{L_P}\right)^2 \sim M^2$$

($GM = R_S$, $1/T_{BH} = dS/dM = R_S/h$)

to be contrasted with previous

$$S_{st} = \frac{M}{M_s} = \frac{L}{l_s}$$

$S_{st}/S_{BH} > 1$ @ small M, $S_{st}/S_{BH} < 1$ @ large M

Where do the two entropies meet? Obviously at

$R_S = l_s$ i.e. at $T_{BH} = M_s$!

"string holes" = states satisfying this entropy matching condition

Using string unification @ the string scale,

$$(L_P/l_s)^2 = g_s^2 \sim \alpha_{GUT}$$

entropy matching occurs for (last eqn. only @ D=4)

$$M = M_{sh} \equiv g_s^{-2} M_s = g_s^{-1} M_P$$

and the common value of S_{st} and S_{BH} is simply

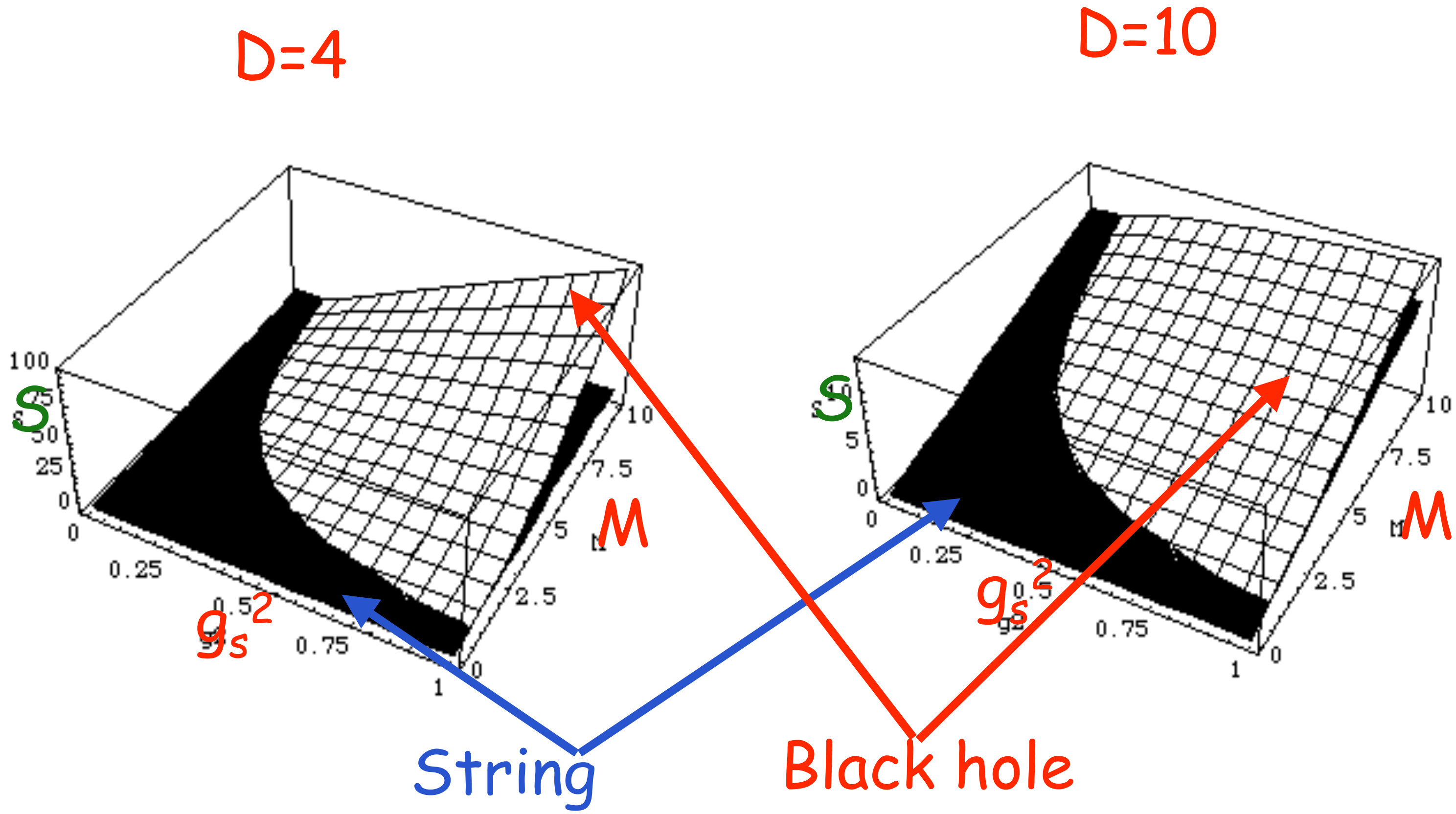
$$S_{sh} = g_s^{-2} \sim \alpha_{GUT}^{-1}$$

In string theory g_s^2 is actually a field, the dilaton.

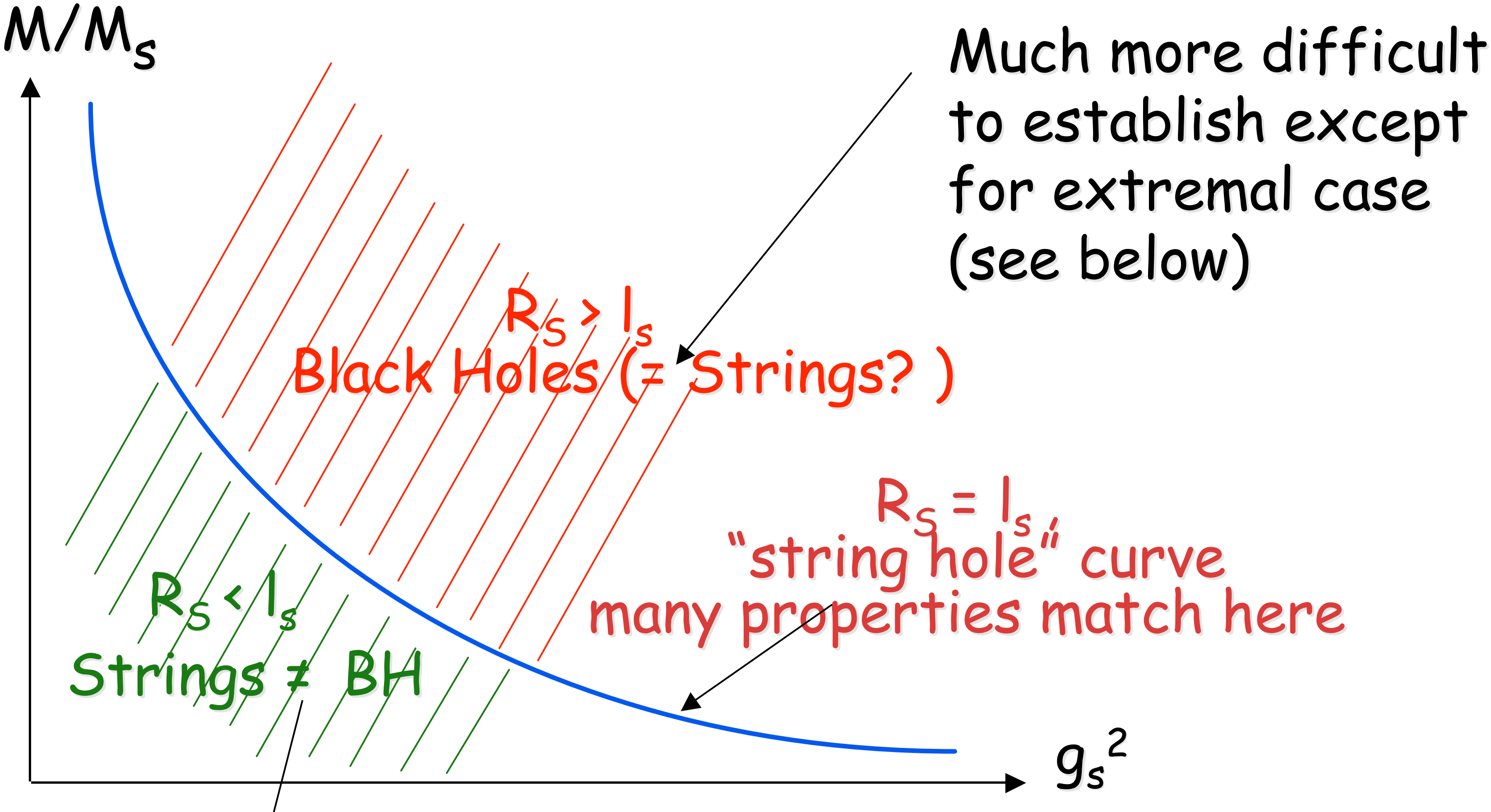
Its value is arbitrary in perturbation theory

Consider the (M, g_s^2) plane

Comparing entropies in D=4, 10



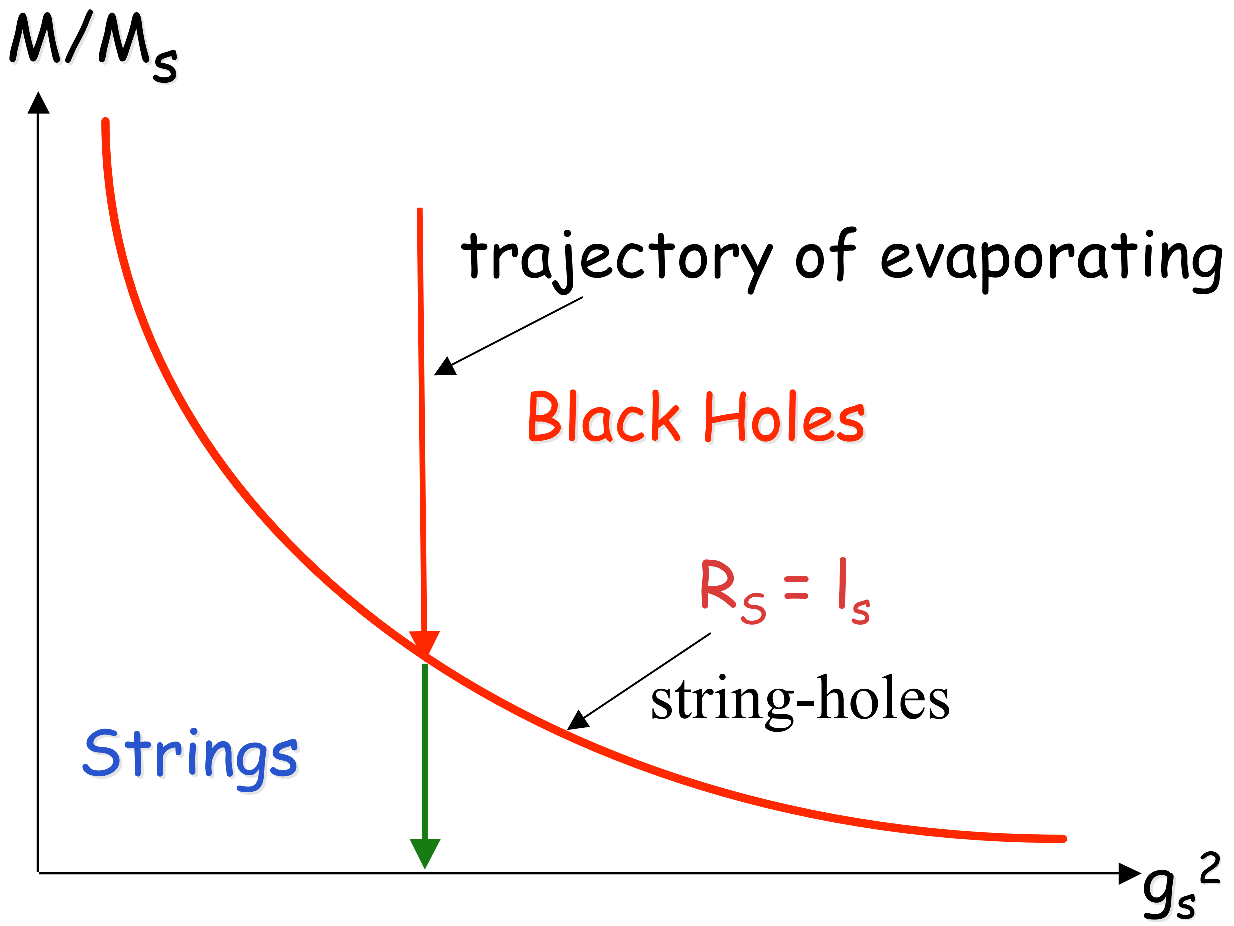
The correspondence curve



Safe conclusion since these strings are larger than R_S

Evaporation at fixed g_s or how to turn a BH into a string (Bowick, Smolin,.. 1987)

Is singularity at the end of evaporation avoided thanks to l_s?



Matching entropy for extremal Black Holes

A. Strominger and C. Vafa, PLB 379 (96);A. Sen, MPL, A10 (95)

C. Callan and J. Maldacena (96)

One takes supersymmetry-preserving (BPS) black-hole solutions in the form of a stack of D-branes possessing certain "charges".

The BH-entropy is known (from the $A/4l_P^2$ formula) as a function of those charges.

At weak coupling (when the D-branes are NOT BHs) one can perform a microscopic counting of the states (excitations of D-branes come from open strings ending on them) and then one uses SUSY to argue that the result can be extended to finite coupling where the D-branes should be BHs.

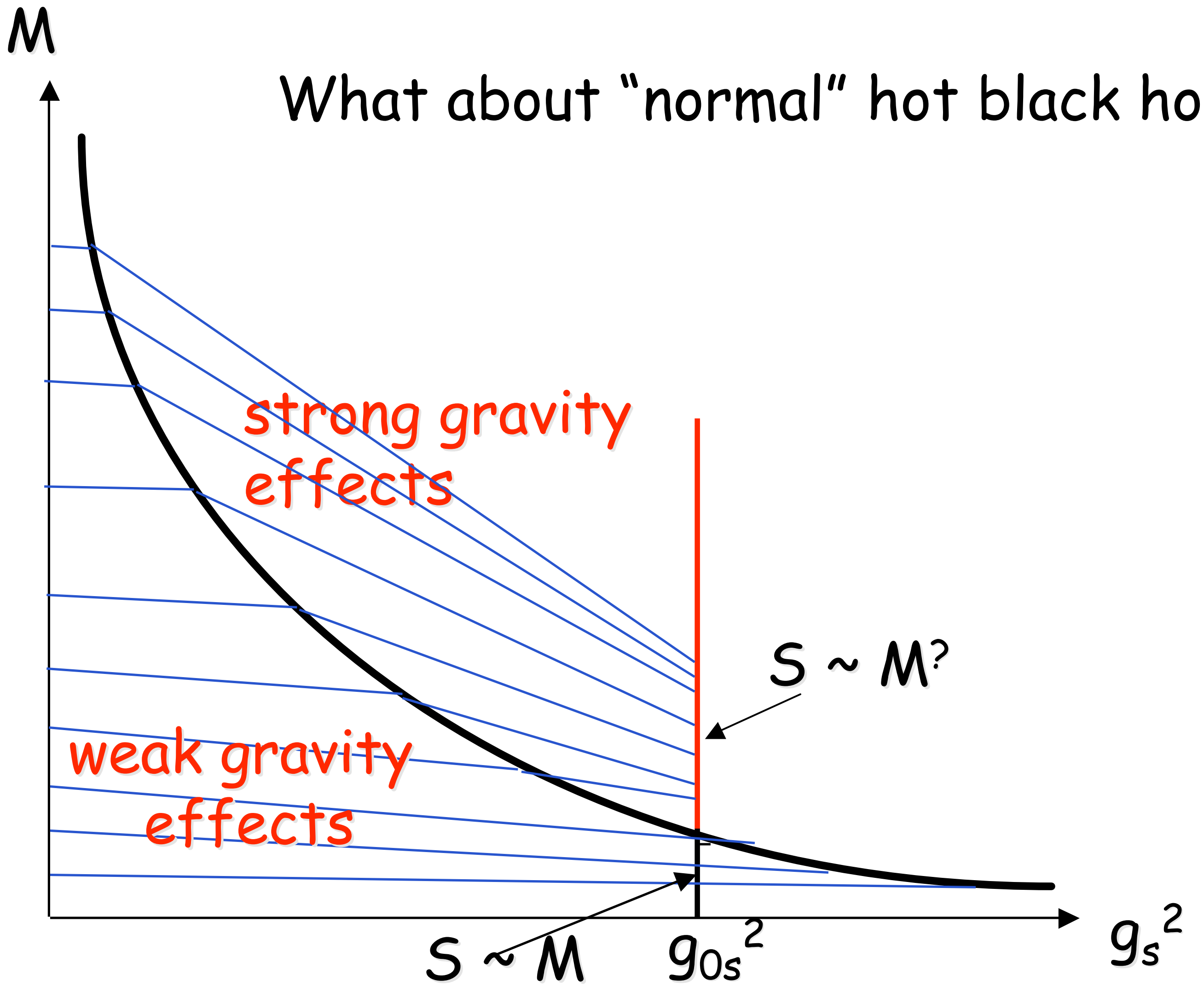
The result matches perfectly the BH formula.

Matching Hawking's evaporation

One can also go a little bit away from the extremal case (BPS black holes are stable) and check the spectrum of emitted quanta. If one averages over the initial D-brane/BH one finds that the emitted quanta obey a thermal distribution with a temperature given by Hawking's formula

This is not the case, at g_s--> 0, if one looks at the decay of individual D-branes. The question of whether the corrections due to a non-vanishing g_s gives BH behaviour for each individual state remains open.

What about "normal" hot black holes?



Matching at (and above?) M_{sh}

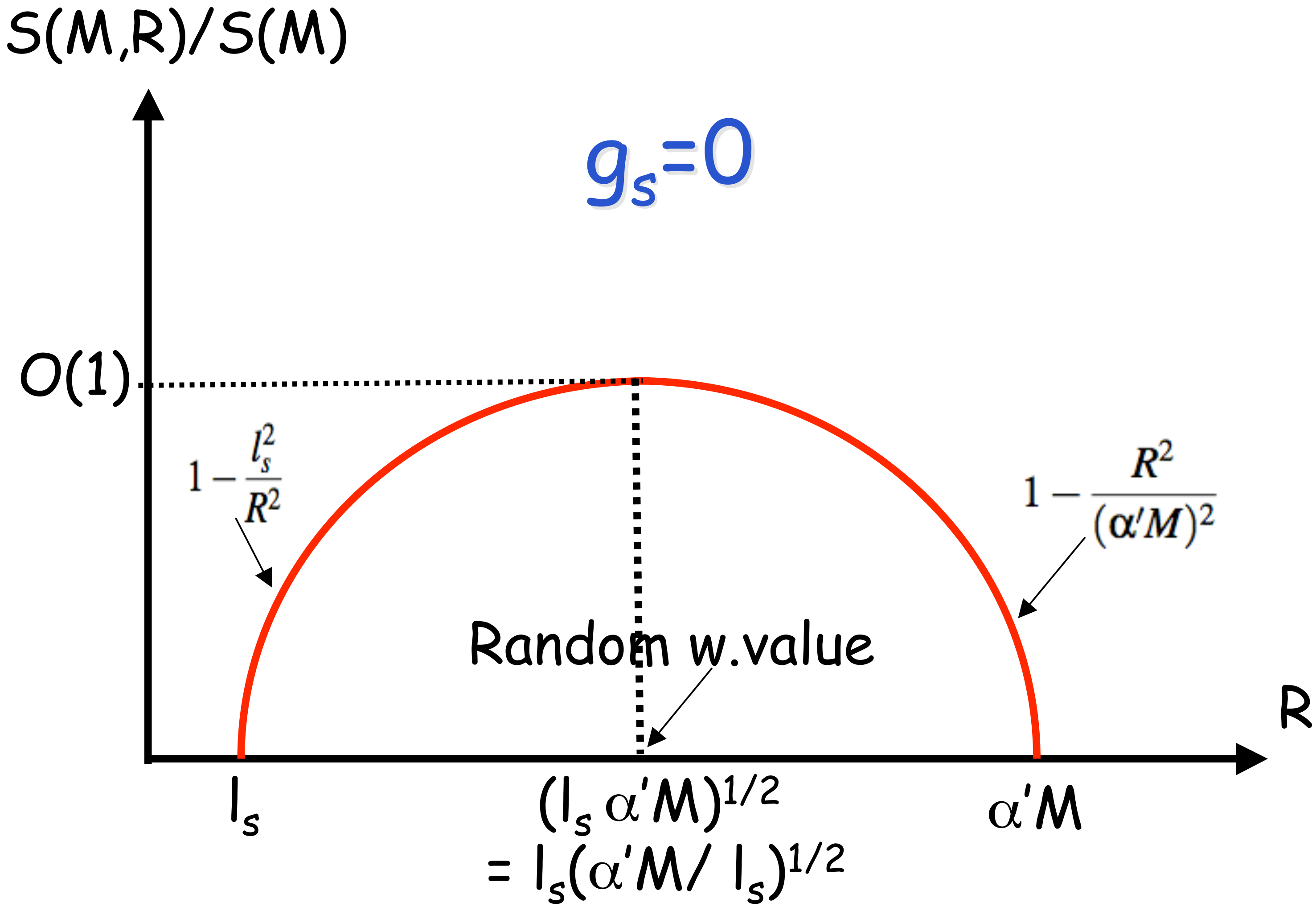
In spite of the naïve matching of their respective entropies, identifying strings and BHs at $M \sim g^{-2} M_s = M_{sh}$ is not obvious. This is because a string of mass M_{sh} is not necssarily contained inside its own Schw. radius $R_S = l_s$ (random walk estimate > l_s)

In order to clarify this issue people have studied the effects of turning on the string coupling, e.g.:

G. Horowitz and J. Polchinski, PRD, 55 ('97); 57 ('98)

T. Damour and G.V. NPB 586 ('00) (hep-th/9907030)

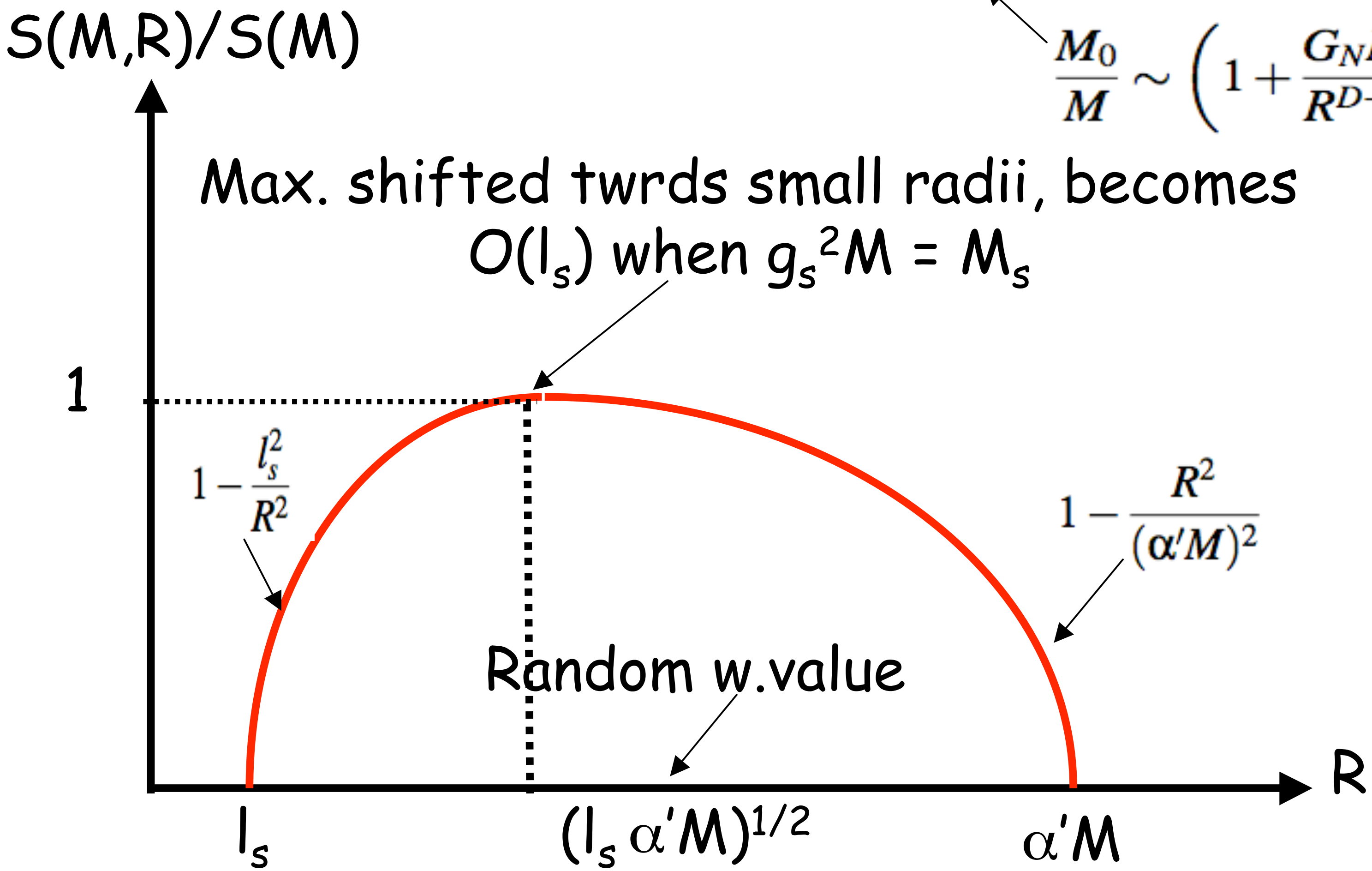
Emerging picture: how is S(M) distributed in R?



$g_s \neq 0$

$$S(M,R) \sim \frac{M}{l_s}\left(1-\frac{l_s^2}{R^2}\right)\left(1-\frac{R^2}{M^2}\right)\left(1+\frac{g_s^2 M l_s^{D-4}}{R^{D-3}}\right) \; ; \; M \equiv \alpha' M$$

$$\frac{M_0}{M} \sim \left(1+\frac{G_N M}{R^{D-3}}\right)$$



Open question: how does the correspondence work above M_{sh}?

$$S(M,R) \sim \frac{M}{l_s}\left(1-\frac{l_s^2}{R^2}\right)\left(1-\frac{R^2}{M^2}\right)\left(1+\frac{g_s^2 M l_s^{D-4}}{R^{D-3}}\right) \; ; \; M \equiv \alpha' M$$

$$\frac{M_0}{M} \sim \left(1+\frac{G_N M}{R^{D-3}}\right)$$

If we use this formula, as it is, for $M >> M_{sh}$ we would get "perfect" agreement for D=4 (at the max. of S) but would actually overshoot BH entropy for D > 4:

$$S \sim M^2 \text{ instead of } S \sim M\, R_S \sim M^{(D-2)/(D-3)}$$

Something must intervene in order to saturate M_0/M at R_S/l_s (by having a minimal R and/or by modifications of the above naïve formula for M_0/M). A nice problem…

Another picture of BH evaporation in ST