Particules Élémentaires, Gravitation et Cosmologie
Année 2006-2007
String Theory: basic concepts and applications

Lecture 6: 16 March 2007

String Cosmology II: Phenomenology

For more details on String Cosmology see, for instance:

- G. Veneziano, Les Houches 1999, hep-th/0002094
- M. Gasperini & G. Veneziano, hep-th/0703055

For even more details see:

- J. Lidsey, D. Wands & E. Copeland, Phys. Rep. 337 (2000) 343
- M. Gasperini & G. Veneziano, Phys. Rep. 373 (2003) 1

Outline

1. Completion/Recap of previous lecture
2. General considerations on cosmological perturbations: qualitative differences between SRI and PBB
3. The amplification of cosmological perturbations during inflation
4. Quantitative differences in SRI and PBB perturbations
5. Four examples:
   - Tensor perturbations
   - Scalar-curvature perturbations
   - EM perturbations
   - Isocurvature axion perturbations and the curvaton mechanism
6. Conclusions

Completion/Recap. of previous lecture

- Why string cosmology?
- Analogy with strong-interaction case.
- AF vs APT, deconfinement transition vs. bounce
- Decoupling of scales

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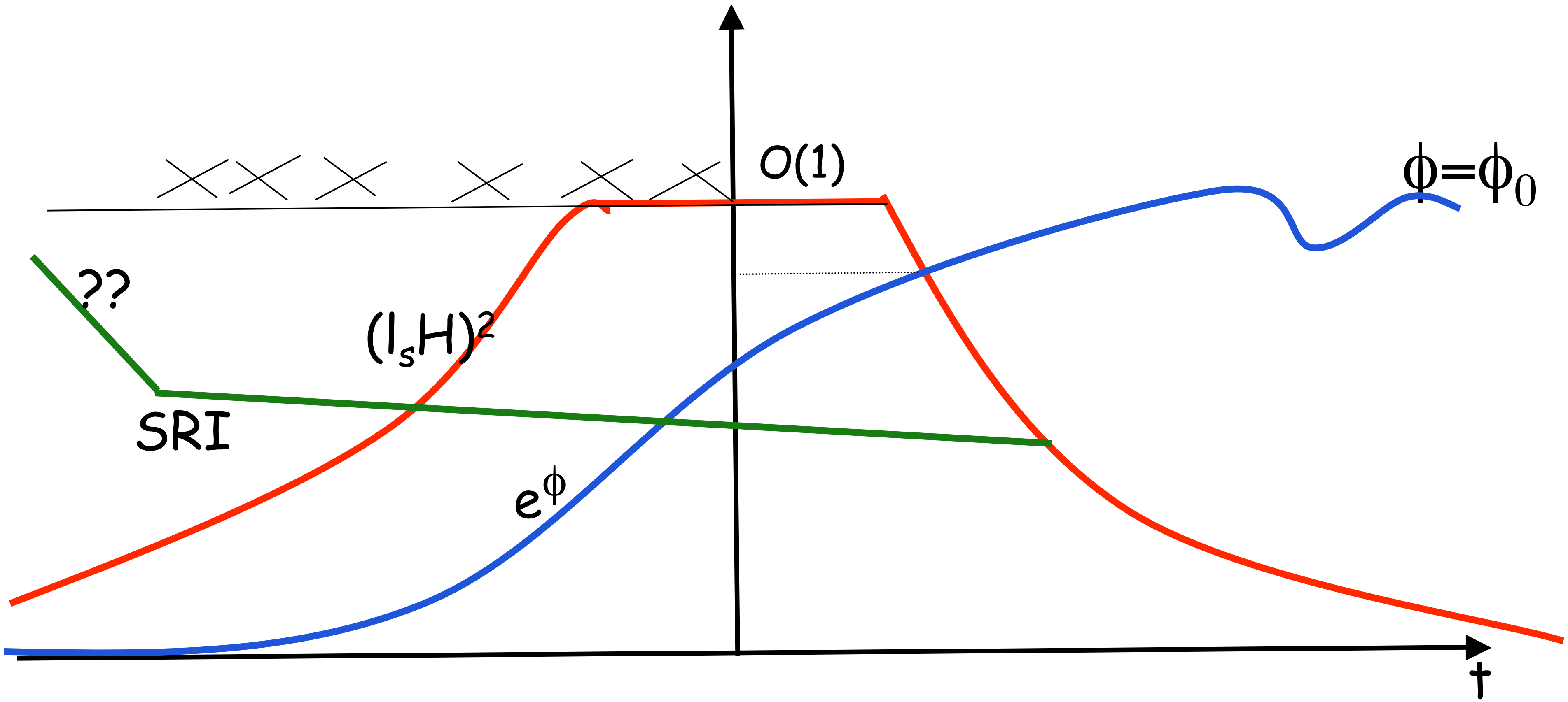
- APT assumption
- Generic emergence of collapse/inflation heading towards singularity/bounce
- Duration of collapse/inflation phase and symmetries in the weak-coupling, small-curvature regime
- Generic emergence of large Universes
- Emergence of isotropic & spatially flat Universes?

Reminder of PBB plot (and SRI)



General Considerations on Cosmological Perturbations

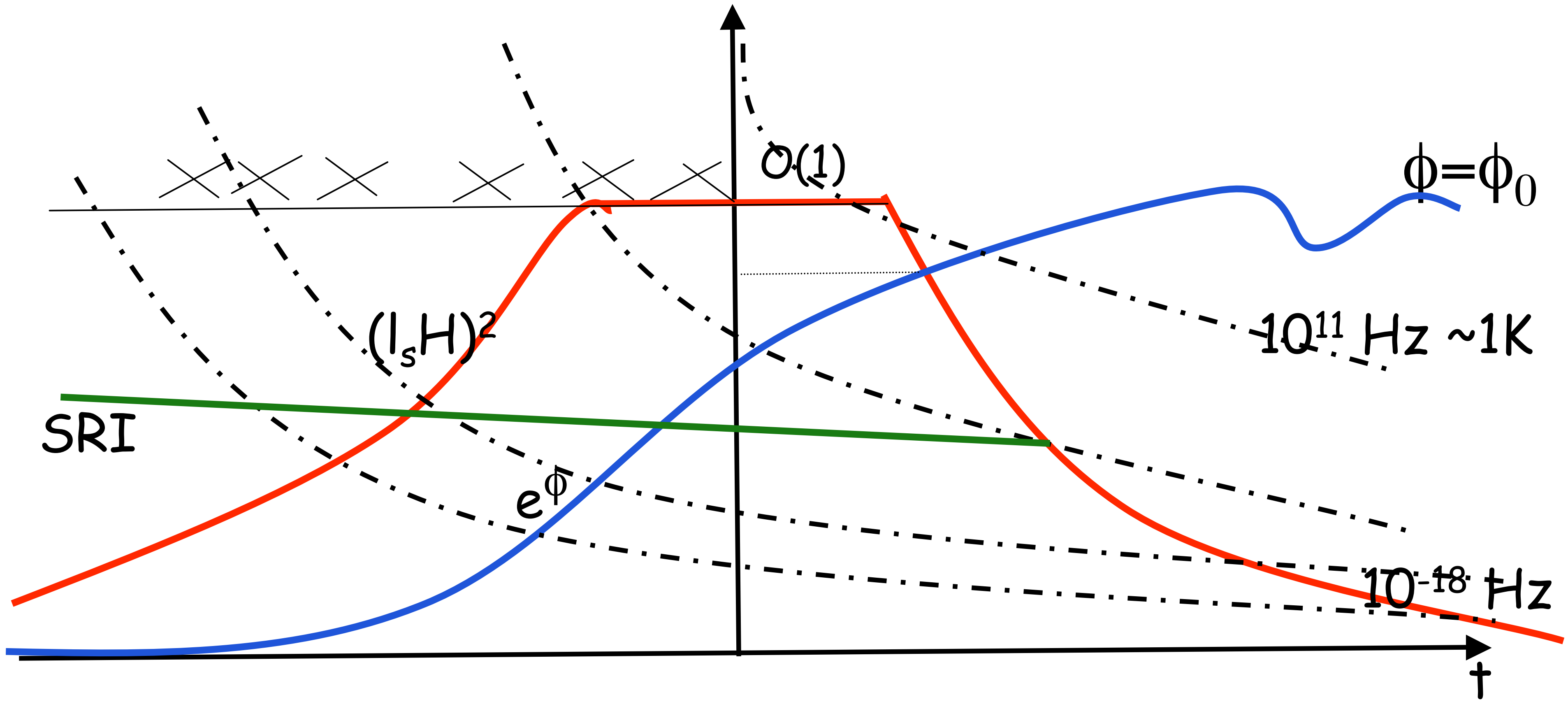
- Physical scales redshift like a(t). By the very definition of inflation, they are pushed outside the horizon during inflation [a(t) grows faster than $H^{-1} = a(t)/(da/dt)$]
- They re-enter the horizon during the FRW phase [a(t) grows less fast than H^{-1}]
- Larger (shorter) scales leave the horizon earlier (later), re-enter later (earlier)

What distinguishes SRI and DDI?

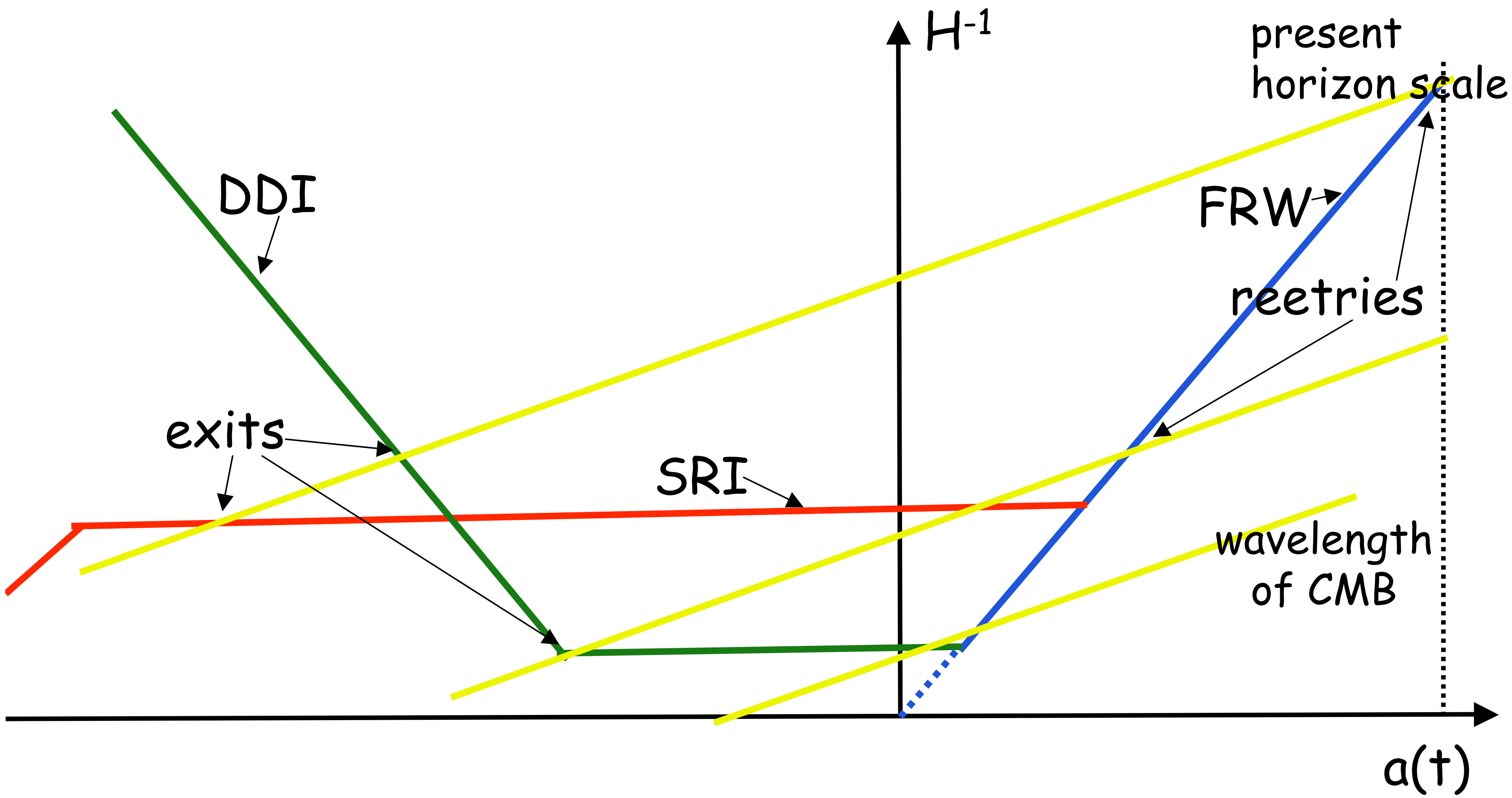
- Since during SRI the Hubble radius is slowly growing, larger scales exit at slightly larger values of H. The opposite is true in bouncing cosmologies
- Because of CMB constraints, H/M_P must be $< 10^{-5}$ at exit of the present Hubble radius. Combined with the previous statement this implies that $H/M_P < 10^{-5}$ at exit of any other observable scale.
- By contrast, in bouncing-curvature cosmologies (e.g. DDI), shorter scales may exit at higher values of H/M_P typically as large as $M_s/M_P \sim 1/10$.

The richer set of backgrounds and fluctuations in string theory allows for a whole set of new phenomena

Reminder of PBB plot w/ different observable physical scales



Kinematics of exit and reentry in SR and DD inflation



Amplification of cosmological perturbations during inflation

- For scales well inside the horizon the cosmological evolution is an adiabatic process
- For scales outside the horizon the cosmological evolution is a sudden process
- There is no particle/energy production from the background in the former case, there is in the latter
- These properties do not depend on the specific cosmology under considerations
- Indeed one can see them even in a toy model: a harmonic oscillator moving in a time-dependent background!

Harmonic oscillator case

$$L = \frac{1}{2} a(t)^2 \left(\dot{x}^2 - \omega^2 x^2\right) \qquad p = a^2 \dot{x} \; ; \; H = \frac{1}{2}\left(a^2 \omega^2 x^2 + a^{-2} p^2\right)$$

Classical eom: $y \equiv ax \; ; \; \ddot{y} + (\omega^2 - \ddot{a}/a) y = 0$

Distinguish two extreme cases:

1. $\omega^2 \gg \ddot{a}/a \qquad y = ax \sim const. \; ; \; q = p/a \sim const.$

H is ~ constant = “adiabatic damping”

2. $\omega^2 \ll \ddot{a}/a \qquad x \sim C_1 + C_2 \int^t a^{-2} + \ldots \; ; \; p = C_2 + \ldots$

x & p frozen:

H grows, dominated by either x or p, depending on sgn(da/dt)!

•These considerations were entirely classical but they have a clear counterpart at the quantum level

•If one starts from the harmonic oscillator in its ground (or a low energy) state it remains there in the adiabatic case: the wave-function adapts to the slowly varying external field

•In the sudden case the harmonic oscillator gets excited into a so-called squeezed state (from quantum optics) in which either $\Delta x \gg \Delta p$, or vice versa. In either case, the h.osc. pumps energy from the external field

•NB: If the "cosmological evolution" lasts a certain time, the total gain in energy depends just on the ratio

$1+z = a_f/a_i$ (the overall "redshift")

From the harmonic oscillator to QFT

For massless scalar fields (and for tensor perturbations) the situation is very similar to that of the H.O. Better work in conformal time η: $g_{\mu\nu} = a^2(\eta)\, \eta_{\mu\nu}$ (for a simple cosm. bkgnd)

$$S = \frac{1}{2} \int d^3x d\eta\, A^2(\eta) \left[\phi'^2 - (\partial_i \phi)^2 \right]$$

where A is some combination of background fields that depends on the particular perturbation one is interested in

One recovers exactly the H.O. structure by going to Fourier space. NB: the Fourier momenta are comoving momenta $k = a\omega$. They do not redshift, have to be compared to things like A'/A ~ a H (which grows during inflation). They exit quite easily..

Typically, each Fourier mode starts in its ground state and, while $k \gg A'/A \sim aH$, remains there with $A\phi \sim$ constant. However, after that scale crosses the horizon ($k < aH$), the corresponding perturbation freezes ($\phi, \Pi \sim$ const.$\sim A^{-1}(ex)$) which amounts to particle production in squeezed states. Example of gravitational waves: $\phi = M_P\, h^{TT}$, $A = a$ (actually a_E)

$$M_P\, \delta h(k) \sim (k/A)_{exit} \sim H(t = t_{ex}(k)) \quad \text{i.e.} \quad \delta h = H/M_P$$

When, eventually, the scale under consideration re-enters the horizon, this amplification becomes a stochastic classical fluctuation of the field ϕ

Again, because of the freeze-out, the final perturbation gives us a picture of what "life" was like when it left the horizon. Many of the details of what went on while it was sup.hor. do not matter (decoupling of large and short-distance physics?)

Perturbations in slow-roll Inflation

- Density/curvature perturbations generated with $n_S \sim 1$ (approximate scale-invariant HZ spectrum)
- Tensor perturbations (GW) generated with $n_T \sim 0$ (also approximately scale-invariant) ,
- T/S = $O(n_T)$, smallish but perhaps observable in CMB polarization, too small for direct GW searches
- Non Gaussian, isocurvature components: small, at least in single-field models
- EM perturbations: absent since inflationary metric couples trivially to Maxwell term (and α is constant)

NB: Scale invariance is a consequence of slow-roll, implying a nearly constant H during inflation.

Perturbations in PBB cosmology

H is now rapidly evolving during pre-bang phase!

- Adiabatic dilaton/curvature perturbations: $n_S=4$?

=> Irrelevant for CMB, LSS

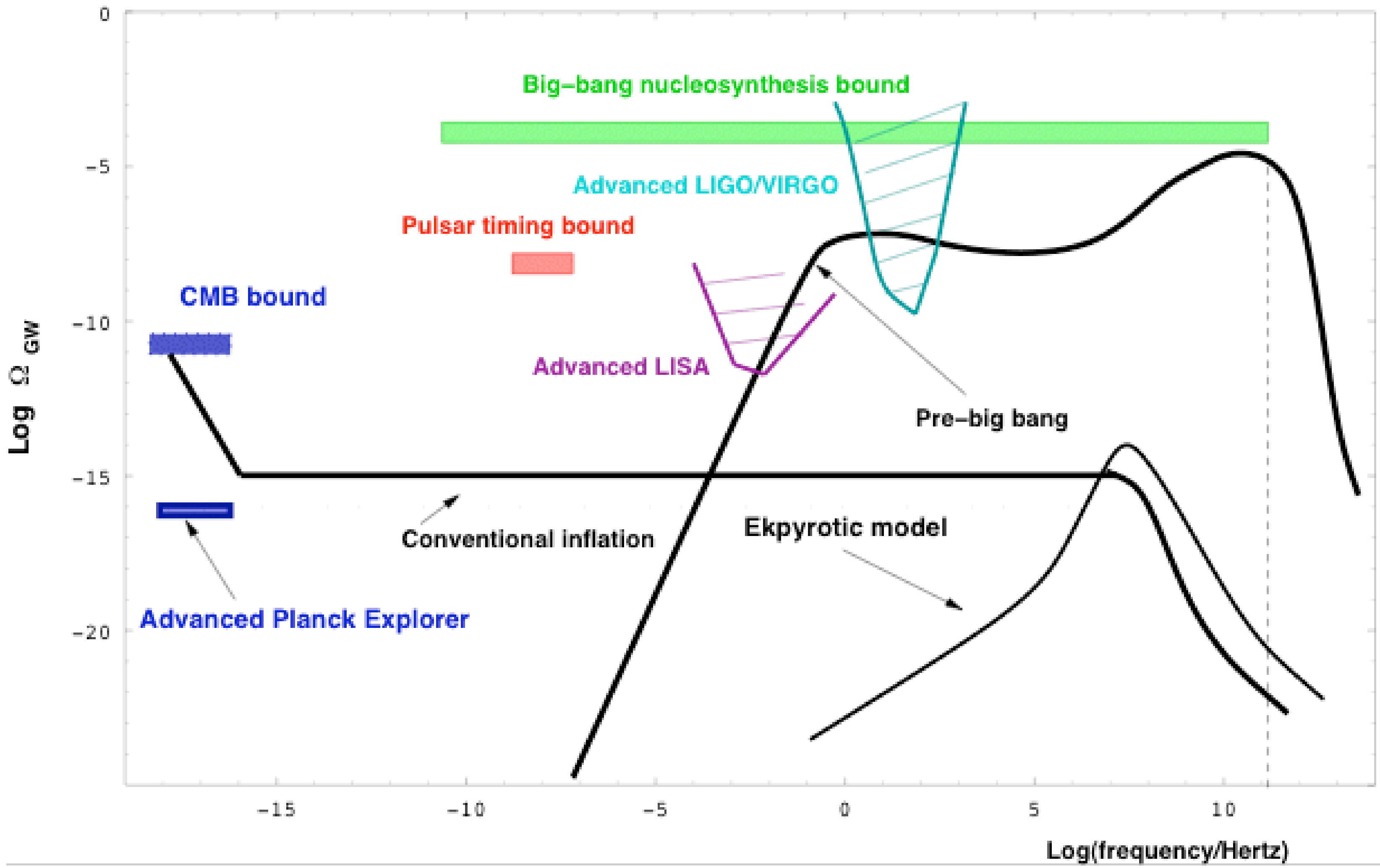
(Ekpyrotic used to claim a scale-inv. spectrum w/out much supporting evidence; recently gave up?)

- Gravitational waves: $n_T = 3$

=> Good for detection, irrelevant for CMB, LSS (see Fig.) (GG, BGGV)

- EM perturbations: amplified due to the time-dep. ϕ, sensitive to evolution of internal space => Seeds for the dynamo of Cosmic Magnetic fields? (GGV)

Gravitational waves: $n_T = 3$



Electromagnetic perturbations

A very characteristic feature of DD cosmologies is that they can also amplify the vacuum fluctuations of the EM field.

The relevant background field A(t) is the effective 4-D fine structure constant $\alpha(t) \sim e^{\phi}/V_6$

For each scale k the amplification depends on the ratio

$$r(k) = \alpha(t_{re}(k))/\alpha(t_{ex}(k))$$

In order to seed the galactic magnetic fields one needs something like $r(k_{gal}) \sim 10^{60}$

This is very hard to imagine in "normal" cosmologies, However, it is all but unnatural in DDI since $(d\phi/dt)$ and $(d\log a/dt)$ are of the same order: => $r(k) \sim (z_{infn})^{O(1)} \sim 10^{O(1)\times 30}$

Q: Where does LSS come from?

A: The universal (Kalb-Ramond) axion of string theory can do the job by playing the role of the so-called curvaton

(Mollerach, Enqvist & Sloth, Lyth & Wands,.. BGGV)

Note: the KR axion of string theory is the SUSY partner of the dilaton: it's there!

- KR-axions: spectrum can be blue, red or flat

$$|\delta\sigma_k|^2 = \left(\frac{H^*}{M_P}\right)^2 \left(\frac{\omega}{\omega^*}\right)^{n-1} \qquad n-1 = \frac{1+3\alpha}{1-\alpha}\ ;\ 3\alpha^2 + \sum_{i=1}^{6} \beta_i^2 = 1$$

$$ds^2 = -dt^2 + \sum_{i=1}^{3} (-t)^{-2\alpha} dx^i dx^i + + \sum_{j=4}^{9} (-t)^{-2\beta_j} dx^j dx^j$$

$$4 - 2\sqrt{3} \sim 0.53 < n < 2 \qquad \text{if } \alpha < 0$$

($H^* \sim M_s$, $\omega^* = H^* a^*/a_0 \sim 10^{11}$ Hz, σM_P = can. field)

n = 1 is right in the (logarithmic) middle!

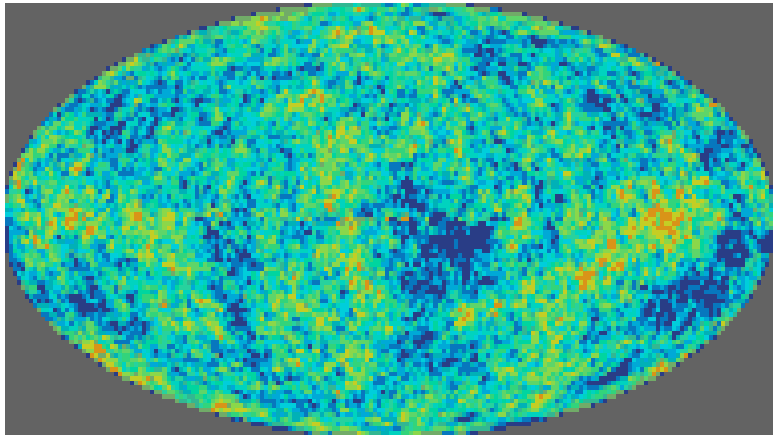
Flat spectrum (n = 1) for symmetric 9-d evolution. A slightly red spectrum, favoured by WMAP, can be easily accomodated

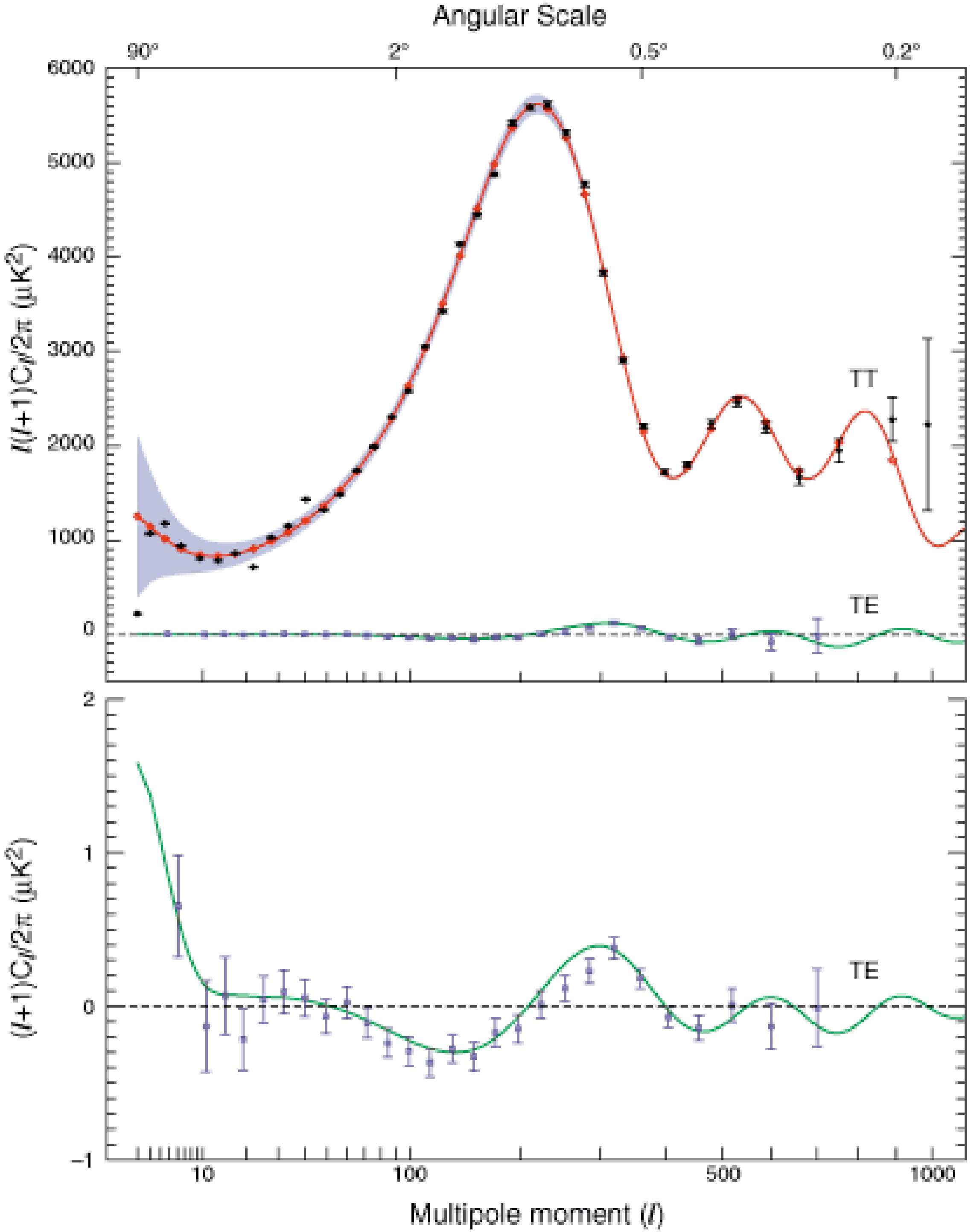
Unlike dilaton, axion does not mix with metric pert.s:

➡ it is an isocurvature (entropy) perturbation;

➡ "wrong" structure of acoustic peaks. That's where the curvaton mechanism comes to the rescue..

WMAP





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- If a V(σ) is generated as the temperature drops and/or the coupling grows, and if <σ> = σ_i is not initially at its minimum, axion pert.s induce calculable curvature pert.s. This "curvaton" mechanism needs:
  - ❶ a phase of axion relevance, dominance.
  - ❷ the axion to decay before NS (m_σ > 10 TeV?)
- Conversion efficiency can be computed. Scalar metric

$$|\Phi_k|^2 = f^2(\sigma_i)\Omega_d^2|\delta\sigma_k|^2 = f^2(\sigma_i)\Omega_d^2\left(\frac{H^*}{M_P}\right)^2\left(\frac{\omega}{\omega^*}\right)^{n-1}$$

where $f(\sigma_i) \sim (4\sigma_i)^{-1}$ (for $\sigma_i < 1$)

and Ω_d refers to the axion energy density at decay

- One then computes the Sachs-Wolfe contribution to the C_l's (n -> 1 wherever limit is smooth)

$$C_l^{(SW)} = \frac{1}{9\pi} f^2(\sigma_i)\Omega_d^2 \left(\frac{H^*}{M_P}\right)^2 \left(\frac{\omega_0}{\omega^*}\right)^{n-1} \times \frac{\Gamma[l+(n-1)/2]}{\Gamma[l+2-(n-1)/2]}$$

($H^* \sim M_s$, $\omega^* \sim 10^{11}$ Hz $\sim 10^{30}\,\omega_0$, $f(\sigma_i) \sim (4\sigma_i)^{-1}$)

NB: $$\frac{\Gamma[l+\ldots]}{\Gamma[l+\ldots]} \sim \frac{l^{n-1}}{l(l+1)} \Rightarrow l(l+1)C_l \sim l^{n-1}$$

- COBE normalization: C_2 = (1.09 +/- 0.23)10^{-10} gives:

$$(1.09 \pm 0.23)10^{-10} = \frac{1}{54\pi} f^2(\sigma_i)\Omega_d^2 \left(\frac{H^*}{M_P}\right)^2 \left(\frac{\omega_0}{\omega^*}\right)^{n-1}$$

=> acoustic-peaks come out fine provided primordial axion spectrum is nearly flat (n~1)

⇒ Slightly blue spectra (n >1) and/or low (H^*/M_P) preferred

⇒ Tensor contribution to CMB still negligible

Q: Can we play with Ω_d to allow a higher H^*/M_P ?

It turns out that one gains a factor ε^{-1} at the price of generating a $f_{NL} \sim \Omega_d^{-1} \sim \varepsilon^{-2}$

$$\frac{\Delta T}{T} = \left(\frac{\Delta T}{T}\right)_G + f_{NL} \left(\frac{\Delta T}{T}\right)_G^2$$

=> Given bounds on f_{NL} ($O(10^2)$) we cannot gain much on normalization... On the contrary some non Gaussianity is all but unexpected

CONCLUSIONS (lects 5 & 6)

- String Theory is still in its infancy (think how long it took to get from early QED to the SM!). Even more true, of course, for string cosmology!
- If we can draw a lesson from the past, it takes the good mix of experimental information and sound theoretical inspiration in order to succeed
- Cosmology is an area of physics where challenging experimental data and equally challenging theoretical questions coexist
- It looks to me like a perfect arena for making progress on fundamental physics issues