# The Anderson Transition

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# Disorder and Trasport Properties

Behaviour of some materials at small temperatures ($T \approx 0$)

**Metal** $\longrightarrow$ **Insulator**

**ANDERSON** 1958: **Disorder** can change the transport properties (Nobel Prize'77).

Disorder $\rightarrow$ interference of the electronic wavefunction $\psi(x)$ with itself $\rightarrow$ $\psi(x)$ is localized in a small region of the materials $\rightarrow$ insulator behaviour.

# The Anderson Model

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

## Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
*Bell Telephone Laboratories, Murray Hill, New Jersey*
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

- The **system** is composed by a **lattice** randomly distributed in space.
- A **particle** on the site $j$ **has a random-distributed energy** $E_j$ with probability distribution $P(E)dE$ which has a width $W$.
- **Between sites there is an interaction** $V_{jk}$ which transfers the particle from the site $j$ to the site $k$.

# Results and Extension of the Anderson Model

- $V_{jk}$ between two sites $j$ and $k$ falls off at large distances faster than $|j-k|^{-3}$.
- $\langle V \rangle < W$

**There is no transport and the wavefunction is localized around few lattice sites.**

Anderson model describes a very specific system. An extension widely used in practice is the Scaling Theory of Localization.
[Abrahams *et al.* Phys Rev Lett 42 (10) 1979]

**Scaling Theory** $\rightarrow$ The $T = 0$ conductance $G$ of a disordered electronic system depends on its lenght scale $L$ in a **universal manner**.

# The Scaling Hypothesis

Conductance $G \rightarrow$ function of the size $L$

$$G = G(L) \Rightarrow \beta(G) = \frac{d\ln G}{d\ln L}$$

**Ordinary Metallic Region**

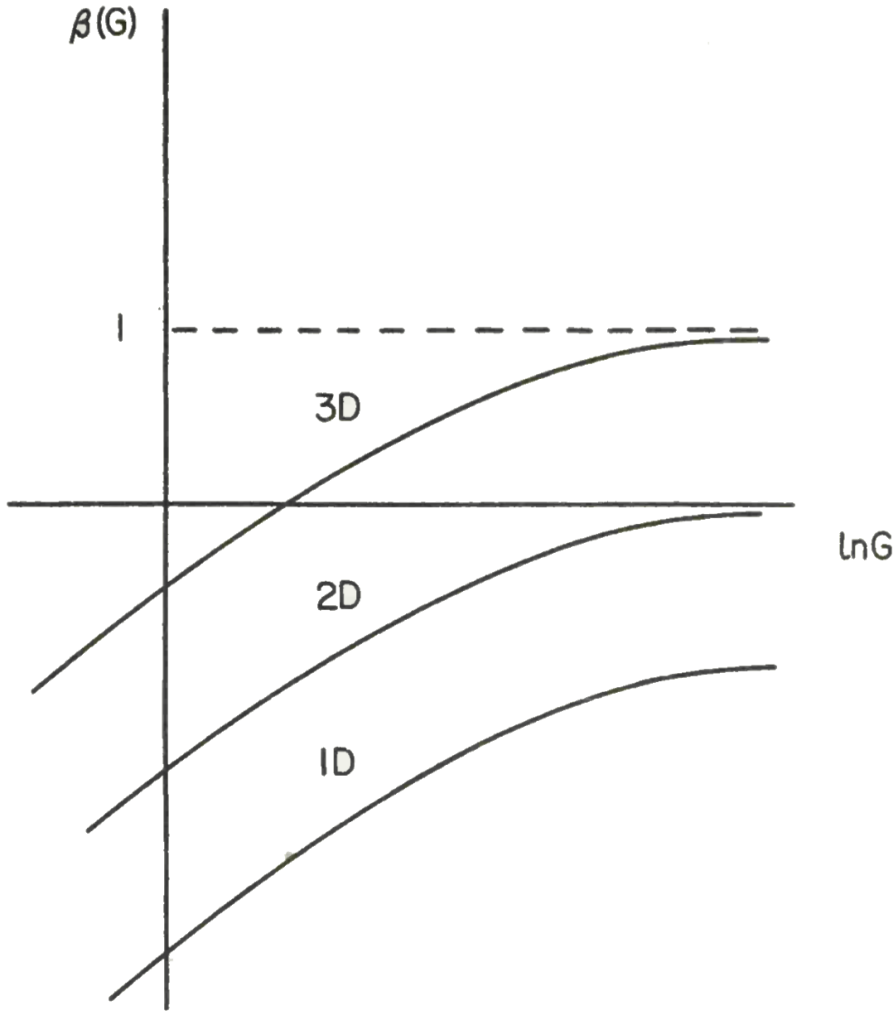
$$G(L) \propto L^{d-2} \Rightarrow \beta(G) = d - 2$$

**With increasing disorder**

$$G(L) \propto e^{-\frac{L}{\lambda}} \Rightarrow \beta(G) = \ln G$$

**Scaling Hypothesis** $\rightarrow$ **Only one universal function** $\beta(G)$ **exists and the equations above are its limiting expressions.**

# More About The Scaling




- $3D \rightarrow \exists\ \hat{G}\ \text{s.t.}\ \beta(\hat{G}) = 0 \rightarrow$ **scaling around fixed point.**

# The Scaling Function

The conductivity at the fixed point is described by

$$\sigma \propto (E - E_c)^{\mu}$$

- $E_c$ is called *Mobility Edge* and is the energy at the critical point.
- $|E| > E_C \rightarrow$ localized states, $|E| < E_C \rightarrow$ extended states.
- For scaling reasons $\mu = \nu(d-2)$ where $\nu$ is the critical exponent of the localization lenght: $\lambda \propto (E - E_c)^{-\nu}$
- Numerical simulations predict $\mu \approx \frac{3}{2}$. Such exponent is the same for all the systems in which an Anderson transition happens.
  [A. MacKinnon and B. Kramer: Z. Phys B 53, **1**, (1983)]

# Experimental Results

# Experimental Details

Consider the Anderson transition inside semiconductors.

Disorder in semiconductors $\rightarrow$ statistical distribution of donor (or acceptors) atoms with concentration $N$ in the host.

How can I change the disorder of the semiconductor?

- **Carrier Concentration**.
- Uniaxial stress $S$.
- Electric/magnetic fileds.

# The Exponent Puzzle

Two types of semiconductor

- Compensated $\rightarrow$ Equal amount of donor and acceptors.
- Uncompensated $\rightarrow$ Not equal amount of donor and acceptors.

**Puzzle:** Experiments report different values for the critical exponent $\mu$ of the conductivity for compensated and uncompensated semiconductors. **Why?**

- **Compensated** $\rightarrow \mu \approx 1.0$
  [U. Thomanschefsky *et al.* Phys Rev Lett 45 13356 (1992)]
- **Uncompensated** $\rightarrow \mu \approx 0.5$
  [Rosenbaum *et al.* Phys Rev Lett 45 1723 (1980)]
  [M.A. Paalanen *et al.* Phys Rev Lett 48 1284 (1982)]

# Possible Solution

VOLUME 71, NUMBER 16 PHYSICAL REVIEW LETTERS 18 OCTOBER 1993

**Possible Solution of the Conductivity Exponent Puzzle for the Metal-Insulator Transition in Heavily Doped Uncompensated Semiconductors**

H. Stupp, M. Hornung, M. Lakner, O. Madel, and H. v. Löhneysen
*Physikalisches Institut, Universität Karlsruhe, D-76128 Karlsruhe, Germany*
(Received 1 June 1993)

The electrical conductivity $\sigma$ (extrapolated to $T=0$) of uncompensated Si:P indicates a crossover as a function of P concentration $N$ at $N_{cr}$ slightly above the metal-insulator transition at $N_c$. For $N > N_{cr}$ the exponent of $\sigma \sim (N-N_c)^{\mu}$ is $\mu \approx 0.64$, while $\mu \approx 1.3$ for $N_c < N < N_{cr}$. At $N_{cr}$ $d\sigma/dT$ changes sign from negative for $N > N_{cr}$ to positive for $N < N_{cr}$. $\sigma$ in a magnetic field also yields $\mu \approx 1$. The apparent discrepancy between uncompensated and compensated semiconductors is traced back to a difference in the (nonuniversal) width of the critical region.

- Conductivity measurements on uncompensated Si:P.
- Disorder is modified changing the concentration of donors.
- The scaling arguments hold for $\sigma(T=0)$
- In the experiment there is an extrapolation of $\sigma(T=0)$ from measures $\sigma(T)$ taken at finite temperature T.

# Conductivity vs Temperature

Electron-electron interactions in disordered systems lead in lowest order to a correction $m\sqrt{T}$ to $\sigma(0)$.

$\sigma(T)$ is measured for various $T$ and $\sigma(0)$ is obtained with the function $\sigma(T) = \sigma(0) + m\sqrt{T}$. **Achtung!**: The MIT is possible only if $\frac{d\sigma}{dT} > 0 \Rightarrow \sigma$ decreases with decreasing $T$.

**PROBLEM**: There is a critical concentration $N_c$ at which a crossover occours and the coefficient of the temperature $m$ changes sign!

$\rightarrow$ The critical region for the scaling is limited for concentrations $N < N_c$

# The Scaling Region

In the experiments with uncompensated semiconductors ($\mu \approx 0.5$) the scaling was performed in the wrong region.

When the concentration of donors is higher than $N_c$ there is no phase transition and and the extrapolation of $\sigma(T = 0)$ is not true.

**Technical Difficulty**: The region in which the scaling is valid is very small. In the experiments is difficult to obtain a good number of data points with the right concentration $N$.

# Experimental Results part 1

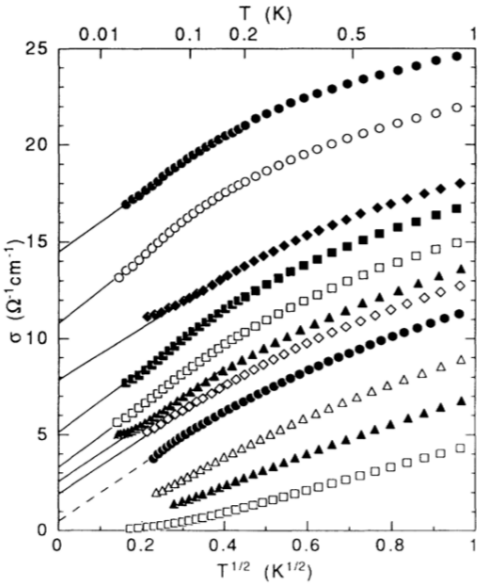



FIG. 1. Electrical conductivity $\sigma$ vs square root of temperature $\sqrt{T}$ for Si:P samples with P concentration $N$ close to the MIT. Solid lines indicate extrapolation to obtain $\sigma(0)$. The concentrations are (from top to bottom in units of $10^{18}$ cm$^{-3}$): 3.69, 3.67, 3.63, 3.60, 3.58, 3.56, 3.55, 3.52, 3.50, 3.45, and 3.38.

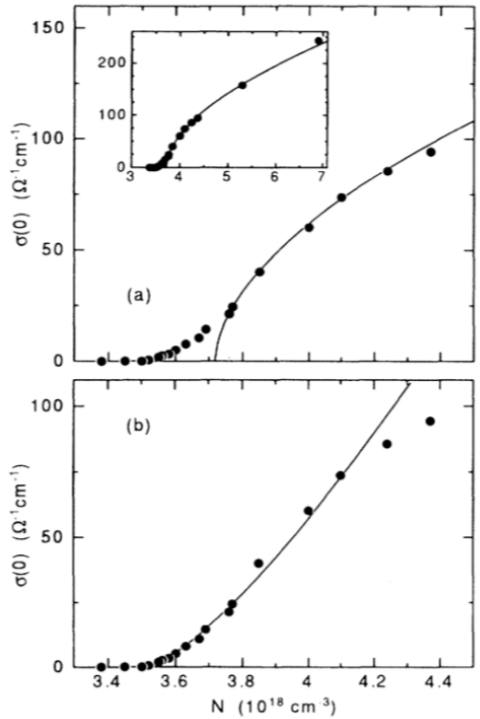



FIG. 2. Extrapolated conductivity $\sigma(0)$ for $T \to 0$ vs P concentration $N$. (a) Fit with $\mu = 0.55$, $N_c = 3.72 \times 10^{18}$ cm$^{-3}$. Inset shows the same fit over an extended $N$ range. (b) Fit with $\mu = 1.3$, $N_c = 3.52 \times 10^{18}$ cm$^{-3}$.

All the graphics are taken from
[H. Stupp *et al.* Phys Rev Lett **71**, 2634 (1993)]

# Experimental Results part 2

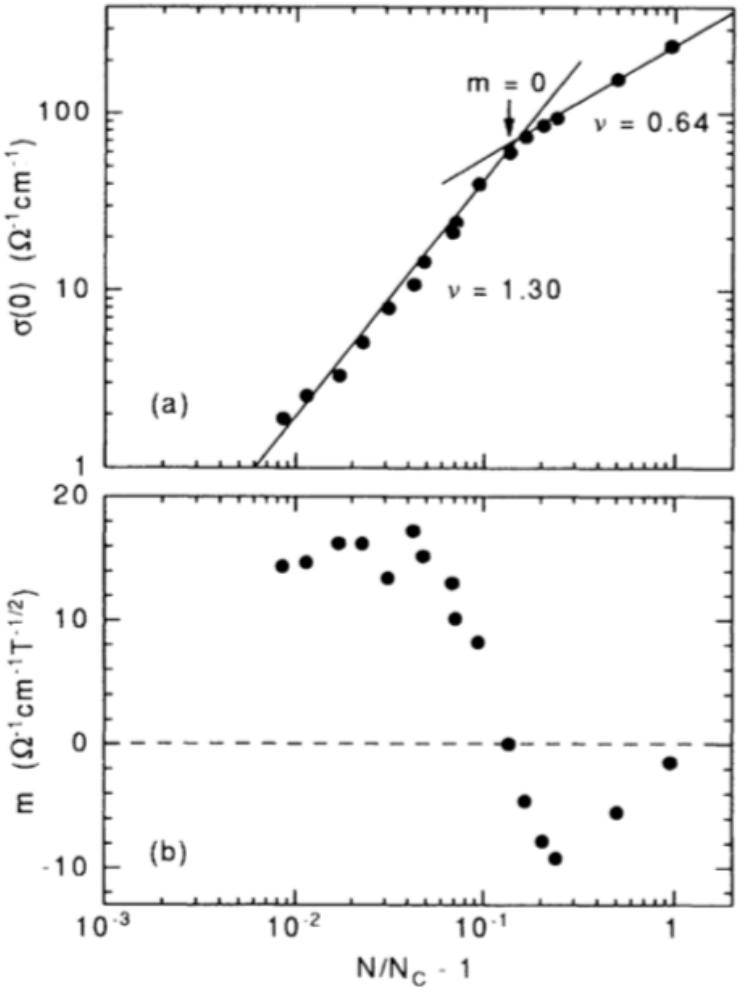



FIG. 3. (a) Extrapolated conductivity $\sigma(0)$ vs reduced P concentration $(N-N_c)/N_c$ with $N_c=3.52\times10^{18}$ cm$^{-3}$. (b) Coefficient $m$ of the $T$ dependence of $\sigma$ vs $(N-N_c)/N_c$.

# Conclusions

- Disorder can localize the electronic wavefunction $\rightarrow$ Metal-Insulator phase transition.
- With scaling arguments we obtain $\sigma \propto (E - E_c)^{\mu}$.
- Numerical simulations predict $\mu \approx \frac{3}{2}$.
- Experiments: compensed semiconductor $\rightarrow \mu \approx 1.0$ while uncompensed semiconductors $\rightarrow \ \mu \approx 0.5$. Why this difference?
- Uncompensated semiconductors $\rightarrow$ the scaling was performed in the wrong region.

# THANK YOU!