QCD properties with external magnetic fields

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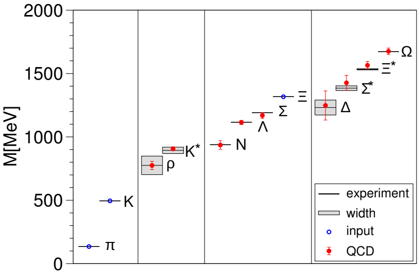
Introduction

- QCD is the theory which describes strong interactions within the Standard Model.

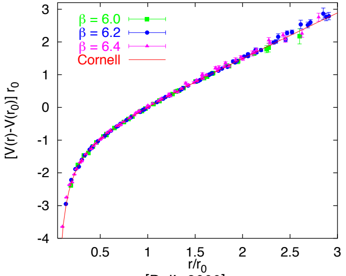
- Perturbation theory works very well in the high energy regime $\rightarrow$ Deep inelastic scattering.

  Problems rise in the low energy regime, perturbative appoach is not allowed $\rightarrow$ Confinement, hadron masses ...

- Lattice QCD $\rightarrow$ first principle approach based on Feynman path integral to solve QCD.



[Durr *et al.*, 2008]

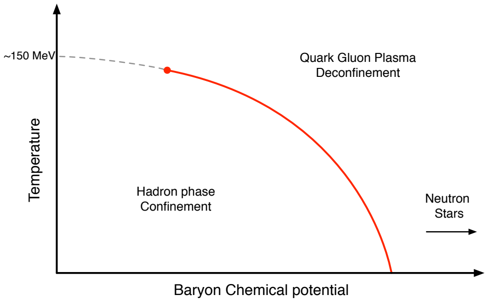


[Bali, 2000]

QCD phase diagram

QCD has a rich phase diagram in the $\mu_B - T$ plane, intensively studied in the recent years:

- $\mu_B = 0$: **analitic crossover** separets hadronic matter and the quark gluon plasma (QGP) (well established).
- Low T and high μ_B: a **first order** transition may be found $\rightarrow$ Neutron stars (still open question).
- If a first order is present, one expect a **critical endpoint** with a second order transition.



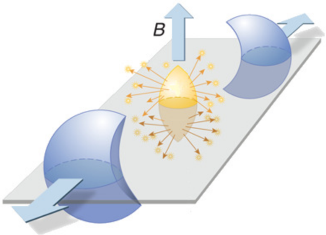
QCD with external B fields

QCD with B fields at the **strong scale**. Found in many phenomenological contexts:

- Neutron stars and compact astrophysical objects, $\mathbf{B} \sim \mathbf{10^{10}}$ **T** [Duncan and Thompson, 1992]
- First phase of off-central heavy ion collisions, $\mathbf{B} \sim \mathbf{10^{15}}$ **T** [Skokov et al., 2009]
- Early universe, $\mathbf{B} \sim \mathbf{10^{16}}$ **T** [Vachaspati, 1991]

We consider the heavy-ion collision scenario:

- Off-central collisions: ions generate magnetic fields, **ortogonal** to the reaction plane. Strength controlled by $\sqrt{s_{NN}}$ and the impact parameter.
- At LHC, B fields expected up to $eB \sim 15 m_\pi^2$



10^{15} Tesla ≈ 0.06 GeV^2
These magnetic fields can lead to relevant modification of the strong dynamics.

QCD with external B fields

Electromagnetic background interacts only with quarks, but loop effects can modify also the gluon dynamics.

- Magnetic field lead to non perturbative effects in:
  - ▷ QCD phase diagram (location of the deconfinament cross over, ...)
  - ▷ QCD vacuum structure (chiral symmetry breaking, ...)
  - ▷ QCD equation of state (effect on the free energy of the QCD medium)

We will discuss non perturbative effects on:

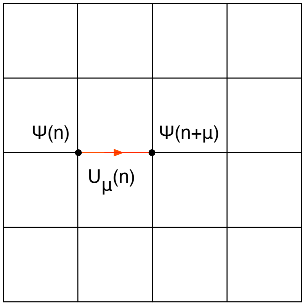
- **QCD equation of state** [PRL 111 (2013) 182001; PRD 89 (2014) 054506]
- **Static quarks potential**. [PRD 89 (2014) 114502]

QCD on the lattice

- Start from path integral formulation of QCD in Euclidean space-time. Discretize the theory over a **finite space-time lattice**. → Regularization

- $\begin{cases} \psi(n),\ \bar{\psi}(n) & \text{quark fields} \\ U_\mu(n) = e^{iagA^a_\mu(n)} & \text{parallel transporters} \end{cases}$

- **Finite number** of integration variables → Monte-Carlo algorithms can be used.



- Sample configurations with the probability distribution: $\det M e^{-S[U]}$, then:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \det M e^{-S[U]} \mathcal{O}[U] \simeq \frac{1}{N} \sum_{i=0}^{N} \mathcal{O}[U^{(i)}] \ .$$

- Temperature of the statistical system: $T = \frac{1}{N_t a}$, with N_t temporal extension.

- Remember: i) check **finite size effects**, ii) perform **continuum limit**.

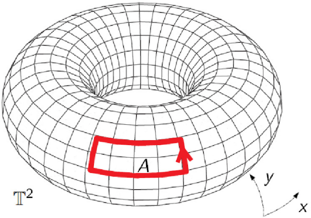
Magnetic fields on the Lattice

- Add proper $U(1)$ phases to $SU(3)$ links:

$$U_\mu(n) \to U_\mu(n) u_\mu(n) \qquad u_\mu = \exp\left(iqa_\mu(n)\right)$$

- Periodic boundary conditions to reduce finite size effects $\to$ **Quantization condition**:

$$e^{iqBA} = e^{iqB(A - L_x L_y a^2)} \to qB = \frac{2\pi b}{L_x L_y a^2}\,, \quad b \in \mathbb{Z}$$

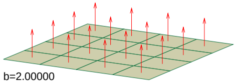


- $\vec{B} = B\hat{z} \to$ gauge fixing $a_y = Bx$, then:

$$u_y^{(q)}(n) = e^{ia^2 qBn_x} \qquad u_x^{(q)}(n)|_{n_x = L_x} = e^{-i\ a^2 qL_x Bn_y}$$

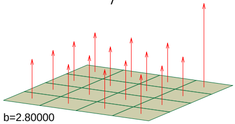
Constant flux $a^2 B$ in all x-y plaquettes, exluded one plaquette at the corner, which has an additional flux $(1 - L_x L_y)a^2 B \to$ Dirac string. Not seen if $b \in \mathbb{Z}$

$b \in \mathbb{Z}$



$b \notin \mathbb{Z}$

- For $b \notin \mathbb{Z}$ string become visible. Non-uniform B $\Longrightarrow$



Free energy density dependence on B

We want to determine $f = f(T, B)$ on the lattice.

- For "small" magnetic fields: $f(T,B) = f(T,0) + \frac{1}{2}c_2(T)B^2 + \mathcal{O}(B^3)$ Then $\chi \propto c_2(T) = \left.\frac{\partial^2 f(T,B)}{\partial B^2}\right|_{B=0}$... **But** $\frac{\partial}{\partial B}$ not defined on the lattice!

- Our method:

  ▷ Analytic extension of $f(T,B)$ (defined only for $B = b \in \mathbb{Z}$) to non-integer B.

  ▷ Calculate on the lattice $\mathcal{M}(T,B) = \frac{\partial f(T,B)}{\partial B}$ (this is **not** the magnetization!).

  ▷ Numerical integration of $\mathcal{M}$ to determine :

$$\Delta f(T,b) = f(T,b) - f(T,0) = \int_0^b \mathcal{M}(B,T)dB \quad b \in \mathbb{Z}.$$

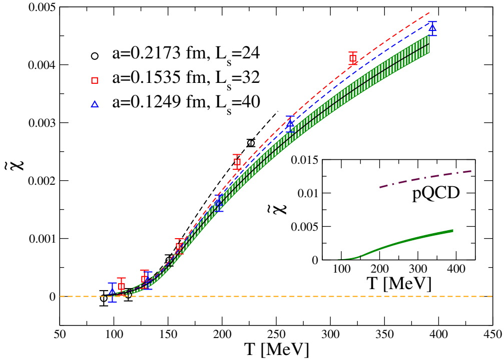
  ▷ B-dependent additive divergences are removed using:

$$\Delta f_r(T,b) = \Delta f(T,b) - \Delta f(0,b) \ .$$

Continuum extrapolation of $\tilde{\chi}$ from our lattice results.

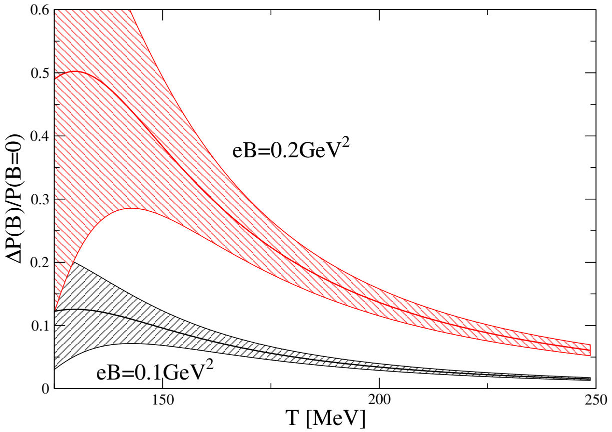
$$\tilde{\chi}(T) = -\frac{e^2 \mu_0 c}{18\hbar\pi^2} L^4 c_2(T)$$

- The QCD medium is a **paramagnet** in all the explored temperature.
- Sharp increase of $\tilde{\chi}$ above $T_C \sim 150 - 160$ MeV.
- Low $T \to$ HRG behavior:
  $$\tilde{\chi}(T) = A\exp(-M/T)$$
- High $T \to$ pQCD behavior:
  $$\tilde{\chi}(T) = A' \log(T/M')$$
- We observed a linear response up to $eB \approx 0.2$ GeV^2.



Free energy density dependence on B

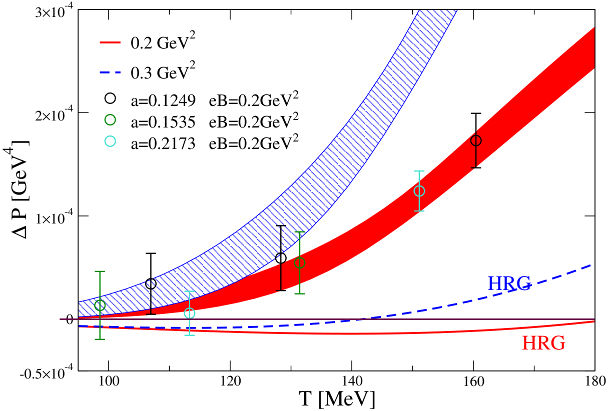
Magnetic contribution to the pressure: $\Delta P(B) = -\Delta f = \frac{1}{2}\tilde{\chi}(eB)^2$.



Of the order 10% for 0.1 GeV^2, 50% for 0.2 GeV^2.

Free energy density dependence on B

Low T: check with the hadron resonance model predictions [Endrődi, 2013]



HRG predicts **diamagnetic** behavior at low-T, as one expects:

$\rightarrow$ Dominant contributions from pions at low T.

- No evidence with present statistics for such behavior.
- Preliminary lattice indications for a diamagnetic behavior up to $T \approx 120$ MeV.

[Bali, Bruckmann, Endrodi et. al., arXiV:1406.0269]

Anisotropic $Q\bar{Q}$ potential

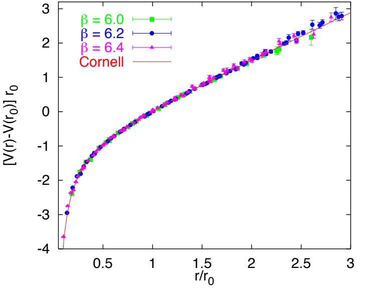
Study of the static potential $V_{Q\hat{Q}}$ with external magnetic fields

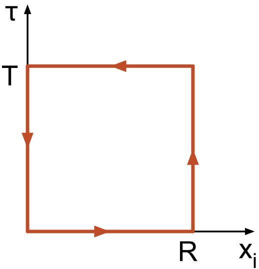
$V_{Q\bar{Q}}$ can be described with the Cornell parametrization:

$$V_{Q\bar{Q}}(|\vec{R}|) = c + \frac{\alpha}{|\vec{R}|} + \sigma|\vec{R}|$$

$\alpha \rightarrow$ Coulomb term
$\sigma \rightarrow$ String tension

- Describes confinement.
- Quarkonium spectrum.





Can be evaluated on the lattice measuring the Wilson loop $W(\vec{r}, T)$:

$$\langle W(\vec{R}, T)\rangle \simeq C \exp\left(-TV_{Q\bar{Q}}(|\vec{R}|)\right)$$

Thus:

$$V_{Q\bar{Q}}(|\vec{R}|) = \lim_{T\to\infty} \log\left(\frac{W(\vec{R}, T)}{W(\vec{R}, T+1)}\right)$$

Anisotropic $Q\bar{Q}$ potential

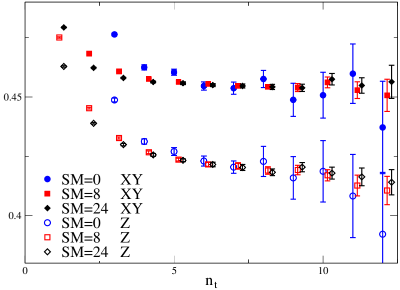
- The introduction of an external field $\mathbf{B} = B\hat{z}$ breaks explicitly the rotation symmetry of the lattice theory.

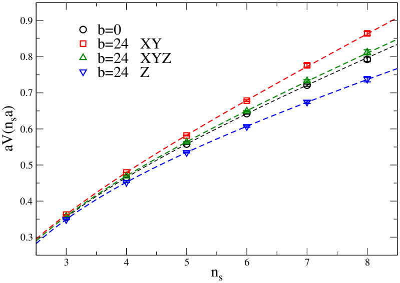
- We calculate $\langle W(\vec{r},T)\rangle$ separating Wilson loops with different spatial orientation.

$$W_{||} = W_Z = W(r\hat{z},T)$$

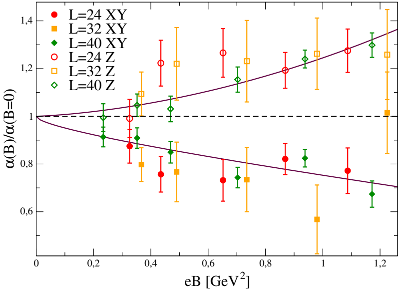
$$W_{\perp} = W_{XY} = \left[W(r\hat{x},T) + W(r\hat{y},T)\right]/2$$

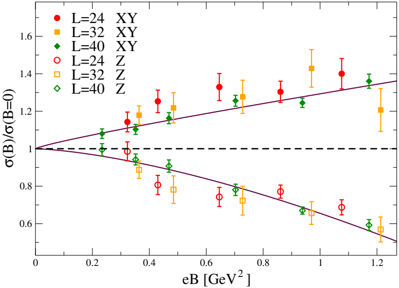
- The obtained potentials $V_{||}$ and $V_{\perp}$ are different





Anisotropic $Q\bar{Q}$ potential





We fit the potential for the different orientations, using the Cornell parametrization:

$$aV(an\hat{d}) = \hat{c}_d + \sigma_d n + \frac{\alpha_d}{n}$$

We define the ratios:

$$R^{\mathcal{O}_d} = \frac{\mathcal{O}_d(|e|B)}{\mathcal{O}_d(|e|B=0)} \qquad \text{where} \qquad \mathcal{O}_d = \hat{\sigma}_d,\ \alpha_d$$

We fit the data with: $R^{\mathcal{O}_d} = 1 + A^{\mathcal{O}_d}(|e|B)^{C^{\mathcal{O}_d}}$

$Obs.$	A^{σ_d}	C^{σ_d}	χ^2/dof
σ_z	-0.34(1)	1.5(1)	0.92
σ_{xy}	0.29(2)	0.9(1)	1.14

$Obs.$	A^{α_d}	C^{α_d}	χ^2/dof
α_z	0.24(3)	1.7(4)	0.32
α_{xy}	-0.24(3)	0.7(2)	1.53

Conclusions

- The QCD medium behaves as a paramagnet in all the explored temperatures.
  - Weak magnetic activity in the confined phase, while the magnetic susceptibility increase sharply across $T_c \approx 150 - 160$ MeV.
  - The QCD medium has linear response up to $eB \approx 0.2$ GeV^2.
  - The magnetic contribution to the pressure is $10 - 50\%$ in the range of fields expected at LHC, $0.1 - 0.2$ GeV^2.

- Anisotropic $Q\bar{Q}$ potential
  - String tension decrease (increase) in the direction parallel (transverse) to the magnetic field, viceversa for the Coulomb term.
  - Anisotropy observed for $eB \gtrsim 0.2$ GeV^2

<u>Future studies:</u>

- Determination of higher order terms $\rightarrow$ relevant for cosmological models, where $eB \sim 1$ GeV^2. Also c quark contributions can be relevant at higher temperatures.

- Heavy meson spectrum modification. Study of anisotropies at finite T $\rightarrow$ Relevant in heavy ion collisions.

BACKUP

Backup

- For small field and a linear, homogeneous, isotropic medium, the magnetization is proportional to the field:

$$\mathbf{M} = \tilde{\chi}\frac{\mathbf{B}}{\mu_0} = \chi\mathbf{H}$$

where $\mathbf{B}$ total field, $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$ external field, and $\chi = \frac{\tilde{\chi}}{1-\tilde{\chi}}$.

- In the small field limit we can use:

$$\Delta f = \int \mathbf{H} d\mathbf{B} \rightarrow \Delta f_r = -\int \mathbf{M} d\mathbf{B} \approx -\frac{\tilde{\chi}}{\mu_0}\int \mathbf{B} d\mathbf{B} = -\frac{\tilde{\chi}}{2\mu_0}\mathbf{B}^2$$

- Our simulations are QED quenched, no backreaction from the medium $\rightarrow$ $\mathbf{B}$ coincides with the external field added to the Dirac operator.

- QED quench does not affect the $\tilde{\chi}$ measure. However, adding the backreaction of the medium increase Δf_R by a factor $1/(1-\tilde{\chi})^2$ $\rightarrow$ Irrelevant a posteriori.

Backup

To get $\Delta f(T, b)$ we measured:

$$\mathcal{M} = \frac{\partial \log Z}{\partial b} = -\left\langle \mathrm{Tr}\left(M^{-1}\frac{\partial M}{\partial b}\right)\right\rangle_b$$

- B no more quantized $\rightarrow$ Oscillations due to Dirac string.
- Numerical integration over $\mathcal{M}$ spline interpolations $\rightarrow \Delta f$
- $\Delta f(B_k, T) \approx \frac{1}{2}c_2(T)B_k^2$. We fit:

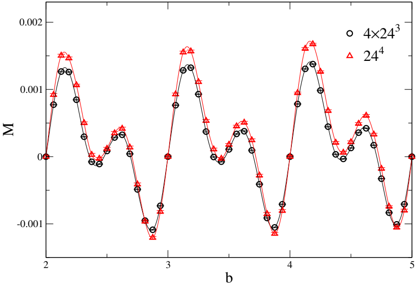
  $$f(b,T) - f(b-1,T) = \int_{b-1}^{b} \mathcal{M}(B,T)dB$$

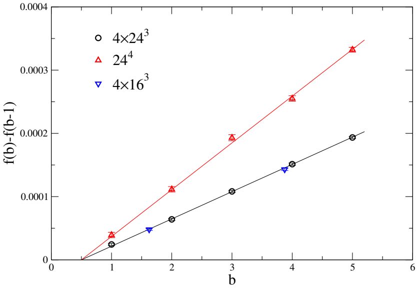
  using:

  $$c_2(T)[b^2 - (b-1)^2] = c_2(T)(2b-1)$$

- $c_2(T)$ determined from linear fit coefficient. Then:

$$\tilde{\chi}(T) = -\frac{e^2\mu_0 c}{18\hbar\pi^2}L^4 c_2(T)$$





Backup

- Temperature: [Zhao and Rapp, '11]
  Assumption for the deconfinement temperature: $T_c \simeq 170$ MeV

T	RHIC at 200 GeV	LHC at 2.76 TeV
$T > 2T_c$	-	$\tau_f < \tau < 1\, fm/c$
$T_c < T < 2T_c$	$\tau_f < \tau < 3\, fm/c$	$1\, fm/c < \tau < 6\, fm/c$
$T = T_c$	$3\, fm/c < \tau < 5\, fm/c$	$6\, fm/c < \tau < 9\, fm/c$

- Magnetic Field:

  eB time evolution @ RHIC for $Au - Au$ collisions for two values of $\sqrt{s_{NN}}$.

  As the collision energy increases the magnetic field increases, but it gets more shrinked in time.

  [Skokov, Illarionov and Toneev, '09]