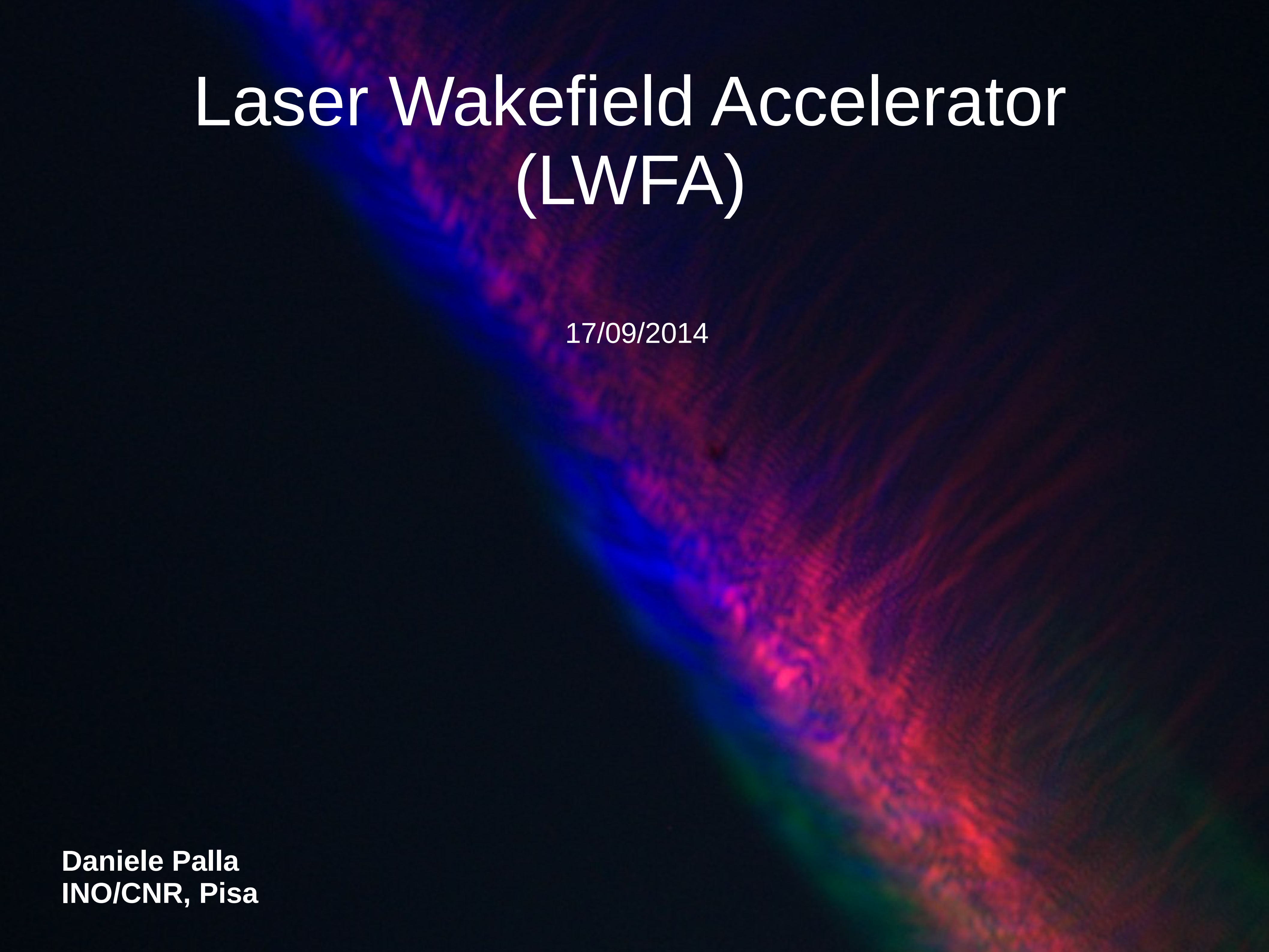

Laser Wakefield Accelerator (LWFA)

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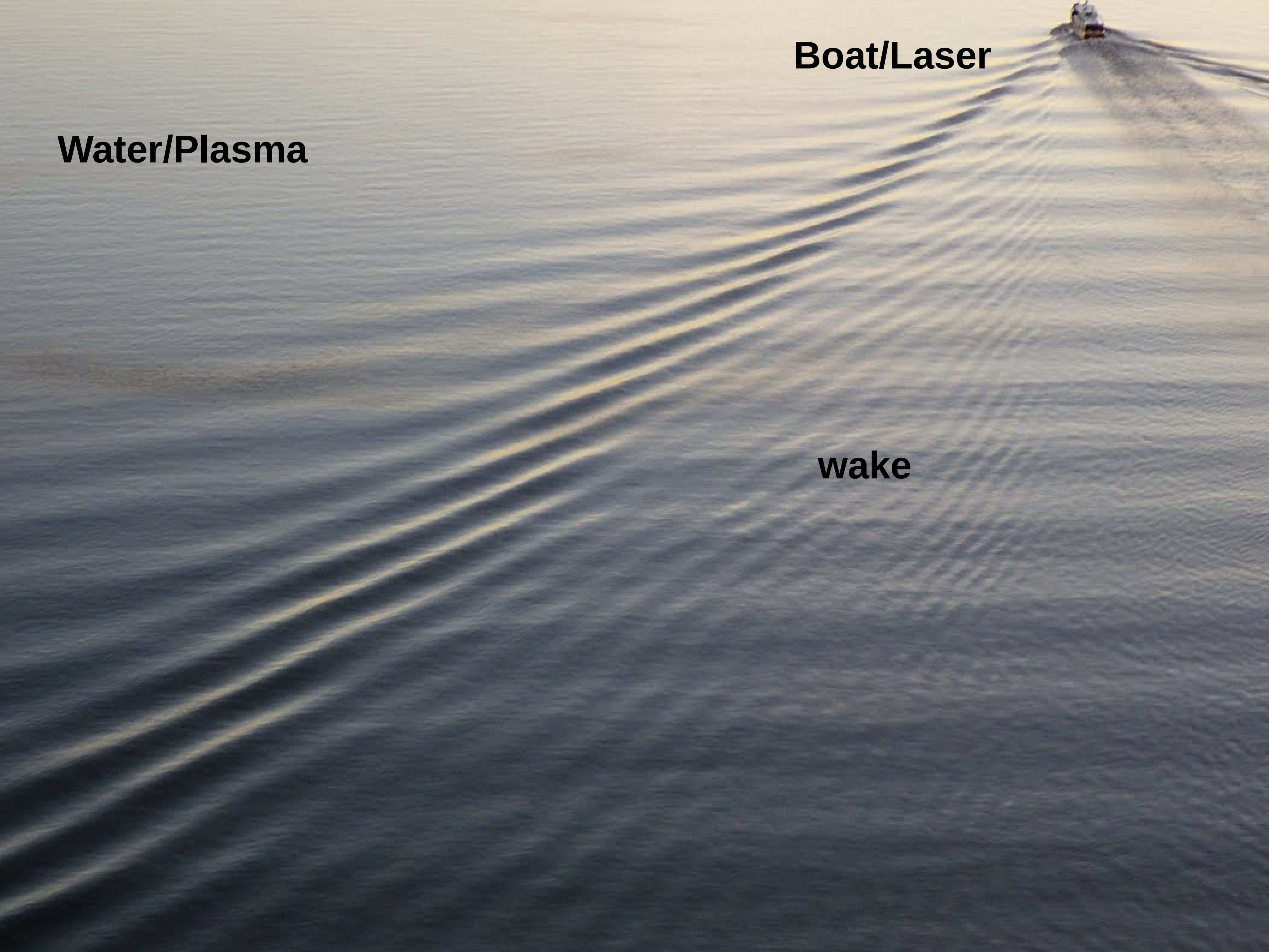

Boat/Laser
Water/Plasma
wake

Electron acceleration

$$\frac{d\vec{P}}{dt}=e\left(\vec{E}+\frac{\vec{v}}{c}\wedge\vec{B}\right)=-\frac{e}{c}\left(\frac{d\vec{A}}{dt}-\vec{v}\cdot\left(\vec{\nabla}\vec{A}\right)\right)$$

Relativistic equation of motion for an electron in EM field

$$\frac{d}{dt}\left(\gamma m_e c^2\right)=-e\vec{E}\cdot\vec{v}$$

$$\vec{A}\left(x,t\right) \qquad \hat{x}\cdot\vec{A}\left(x,t\right)=0$$

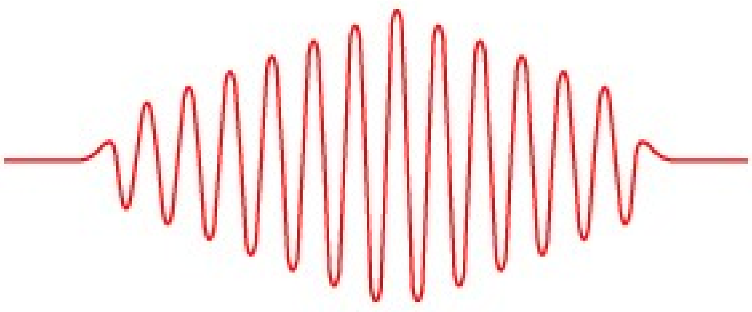
Plane wave

$$\vec{P}_{\perp}=\frac{e}{c}\vec{A} \qquad P_x=\frac{1}{2m_e c}\left(\frac{e\vec{A}}{c}\right)^2$$

“No acceleration theorem”

Ponderomotive force

$$\vec{E}(\vec{r},t)=\Re\left(\tilde{\vec{E}}\underbrace{(\vec{r},t)}_{slow}\underbrace{e^{-i\omega t}}_{fast}\right)$$



In many realistic situation we can assume the existence of two very different time scale: time pulse and oscillation period T.

$$\left\langle m_e \frac{d\vec{v}}{dt}\right\rangle_T = -\frac{e^2}{2 m_e \omega^2}\vec{\nabla}\left\langle E^2\right\rangle_T$$

Ponderomotive force (PF) doesn't depend of particle charges (+-e). PF effect on ions is negligible.
The main effect of PM force is that electron will be expelled from the region were the electric field is higher.

waves

$$\partial_t \vec{U}_e = -\frac{e}{m_e}\vec{E}$$

$$\omega_p^2 = \frac{4\pi e^2 n_e}{m_e}$$ Plasma frequency

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = \left(\frac{\omega}{c}\right)^2 \underbrace{\left[1-\left(\frac{\omega_p}{\omega}\right)^2\right]}_{\epsilon(\omega)}\vec{E}$$

$$n^2(\omega) = \epsilon(\omega)$$ Refractive index

$$\left[-k^2 + \left(\frac{\omega}{c}\right)^2 \epsilon(\omega)\right]\vec{E} = 0 \;\rightarrow\; \omega^2 = c^2 k^2 + \omega_p^2$$

Transverse waves

$$\epsilon(\omega) = 0 \;\rightarrow\; \omega = \omega_p$$

Longitudinal waves

In longitudinal waves, the wavevector k is not determined by wave equation.
This type of wave are called “plasma waves”

Underdense and overdense plasma

$$E \sim e^{ikx - i\omega t}$$

The propagation of the wave requires k = |k| to be a real number, which occours when $\omega > \omega_p$. For a given frequency ω this condition can be also written as a condition on the plasma density

$$n_e < n_c \equiv \frac{m_e \omega^2}{4\pi e^2} = 1.1 \times 10^{21}\, cm^{-3} (\lambda / 1\mu m)^{-2}$$ Underdense plasma

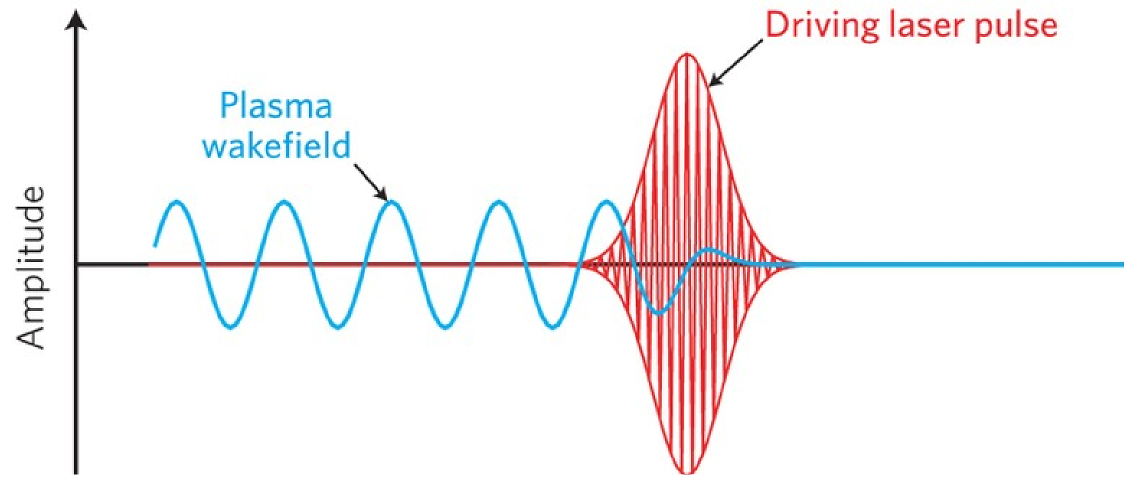
For typical laser wavelength (800-1000 nm), gases are usual underdense. Otherwise, when , $n_e > n_c$ plasma said overdense. In this case the wavevector k became imaginary and we can define a penetration leight:

$$l = \frac{c}{\sqrt{\omega_p^2 - \omega^2}}$$

$$E \sim e^{-\frac{x}{l}}$$

wakefield

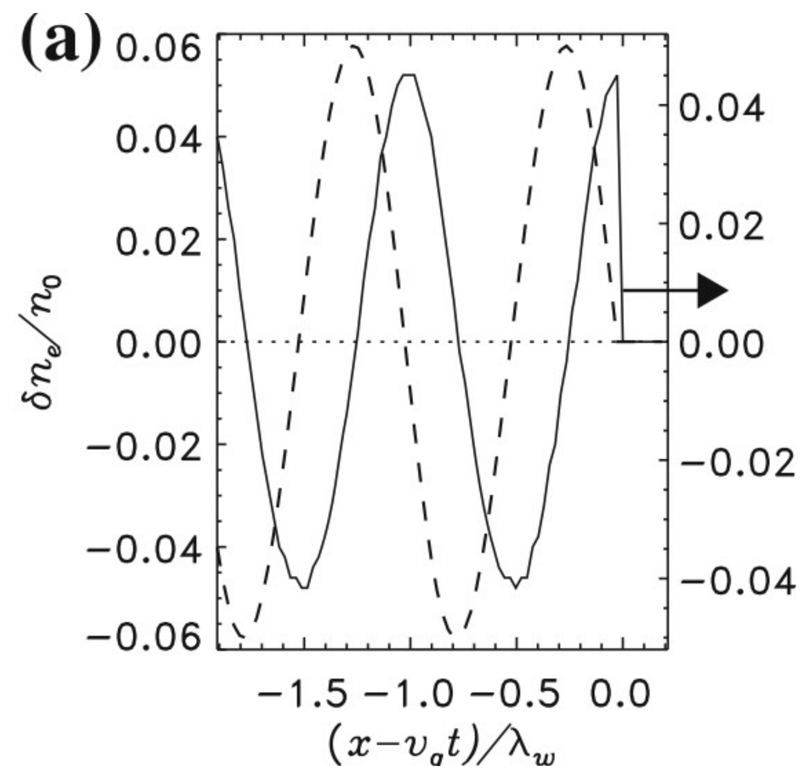
The wavelength and phase velocity of the plasma wave are determined by the way the wave is excited. We can imagine a laser pulse like a traveling (PM) force who generate a wakefield with the same phase velocity of laser pulse.



$$f_{pond} \sim m_e U_o \delta(\underbrace{t - \frac{x}{v_f}}_{\tau}) \rightarrow U_x \sim U_o \cos(\omega_p \tau)\theta(\tau)$$

$$E_x \sim \frac{m_e \omega_p U_o}{e} \sin(\omega_p \tau)\theta(\tau)$$

$$\delta n_e \sim n_0 (\frac{U_0}{v_f}) \cos(\omega_p \tau)\theta(\tau)$$



Wave breaking

$$n_e = n_0 + \delta n_e \;\rightarrow\; |\delta n_e| \leqslant n_0 \qquad |U_x| \leq v_f = v_p \qquad |E_x| \leq \frac{m_e \omega_p v_p}{e}$$

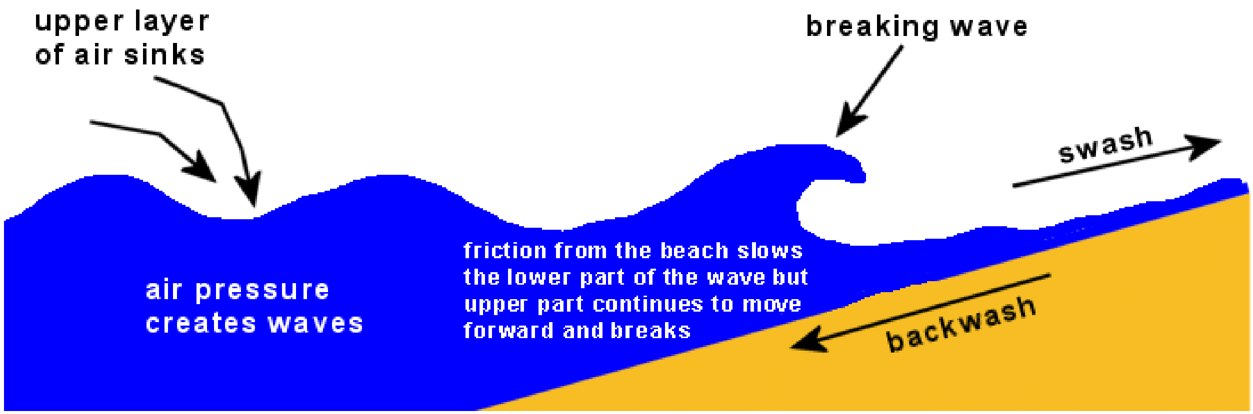
When a longitudinal wave is driven up to such limit, eventually the regular periodic structure is lost, and the wave is said to break. The deformation of the wave depends of the the effect of the nonlinear term:

$$m_e n_e \partial_t \vec{U}_e \;\rightarrow\; n_e \frac{d\vec{P}}{dt} = n_e m_e [\partial_t + \underbrace{(\vec{U}_e \cdot \vec{\nabla})}_{non\,lineare}] (\underbrace{\gamma(\vec{U}_e)}_{nonlineare} \vec{U}_e)$$

The simplest case of wave braking is the brake of gravity waves:

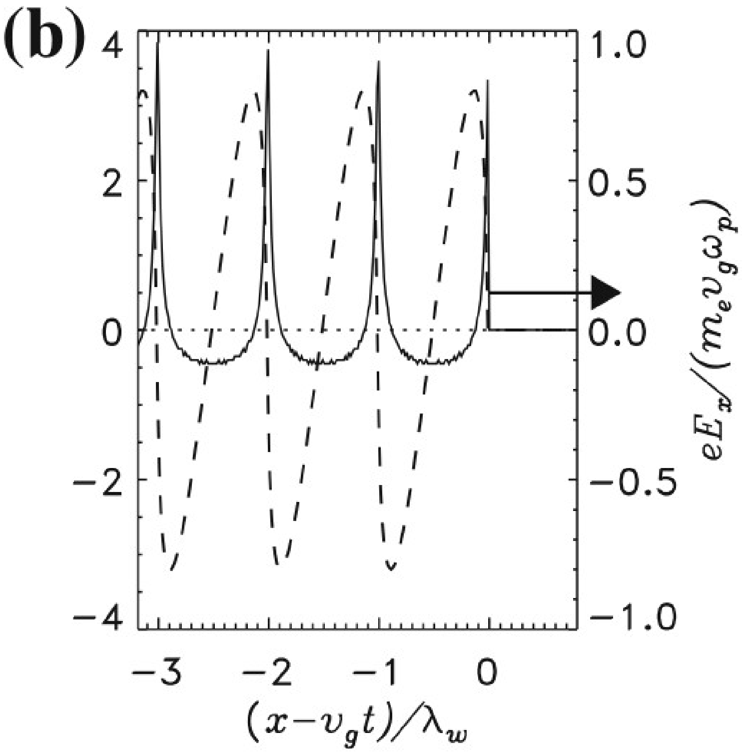
$$\omega^2 = gk\,tgh(kh)$$

$$U_g = \frac{\partial \omega}{\partial k}$$



If we search a solution of the type $E_x(x-v_p t)$ (in a more realistic non linear plasma model) we find:

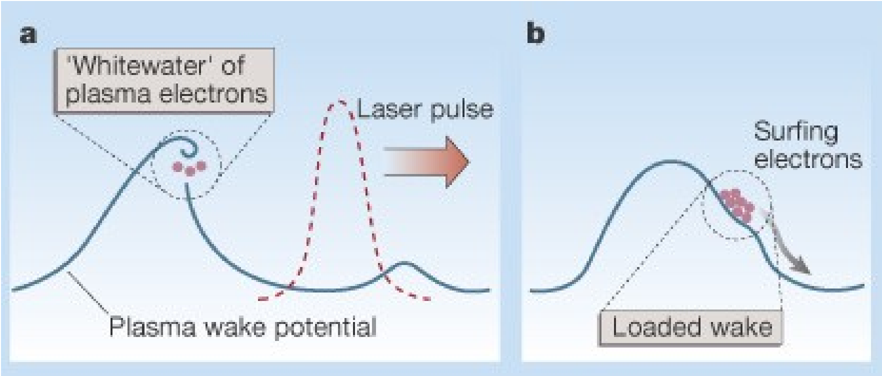
$$n_e=\frac{n_o}{1-\frac{U_x}{v_p}}=\begin{cases} n_o\left(1+\frac{U_x}{v_p}\right) & U_x/v_o \ll 1 \\ singolare & U_x=v_p \end{cases}$$



The maximum speed for the electron is $|U_x|_{max}=v_p$.
The maximum value of the electric field corresponds to $U_x=0$ and $|U_x|_{max}=v_p$.

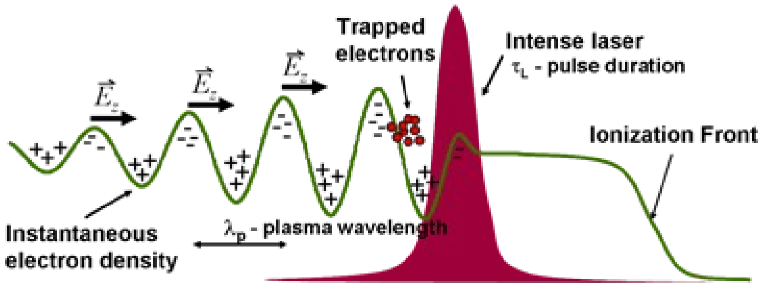
$$E_{max}=\frac{m_e\omega_p c}{e}\sqrt{2(\gamma_{max}-1)}$$

In this model U and E are out of phase but if we were able to “put” an electron (with speed v_p) in the best position (were E is maximum), the electron may be accelerated by the wakefield



LWFA

In a realistic case the pulse duration is not zero but a finite time τ_L . PF force have different signs on rising and falling pulse front. The condition to excited a plasma wakefield is fixed by:



$$\tau_L = \frac{\pi}{\omega_p} \quad \sim \quad 10\, fs$$

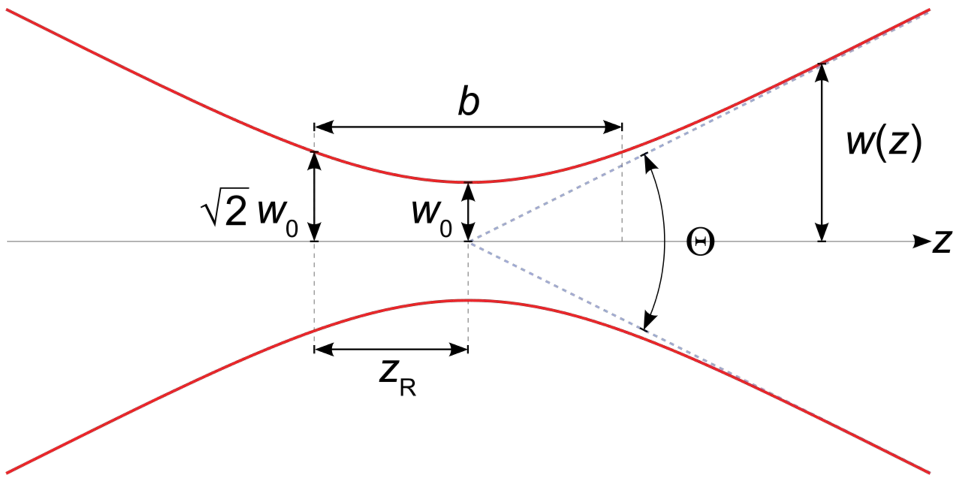
The luckiest electron have U=v_p at a maximum of the potential energy. In this condition the electric field is static (and maximum) in the electron's SR. We can also find:

$$\epsilon \sim 2 m_e c^2 \left(\frac{\omega}{\omega_p}\right)^2 \qquad L_{acc} = \frac{\epsilon}{e E_{max}} = \frac{\lambda}{\pi}\left(\frac{\omega}{\omega_p}\right)^3 \qquad E_{max} = \frac{m_e \omega_p c}{e}$$

$$\epsilon \equiv 100\, MeV \quad \lambda \equiv 1\, \mu m \quad \omega/\omega_p = 10 \quad n_e \sim 10^{19}\, cm^{-3} \quad L_{acc} \sim 300\, \mu m$$

Gaussian optics

Usually we can approximate a laser beam like an ideal gaussian beam .The fundamental model is parameterized by waist, Rayleigh length, ecc.



$$E(r,z)=E_0\frac{w_o}{w(z)}e^{-\left(\frac{r}{w(z)}\right)^2}e^{-i\left(kz+k\frac{r^2}{2R(z)}-\zeta(z)\right)}$$

$$w_0=\left(\frac{\lambda z_0}{\pi}\right)^{\frac{1}{2}}\quad \zeta(z)=\tan^{-1}\left(\frac{z}{z_0}\right)\quad w(z)=w_0\left(1+\left(\frac{z}{z_o}\right)^2\right)^{\frac{1}{2}}\quad R(z)=z\left(1+\left(\frac{z_o}{z}\right)^2\right)^{\frac{1}{2}}$$

All this parameter's value depends by the laser properties and optics (mirror, parabola, ecc). Typical values are:

$$\theta\sim\frac{w_o}{z_0}\equiv 1/10\quad w_0\sim 5\mu m\quad z_o\sim 50\mu m$$

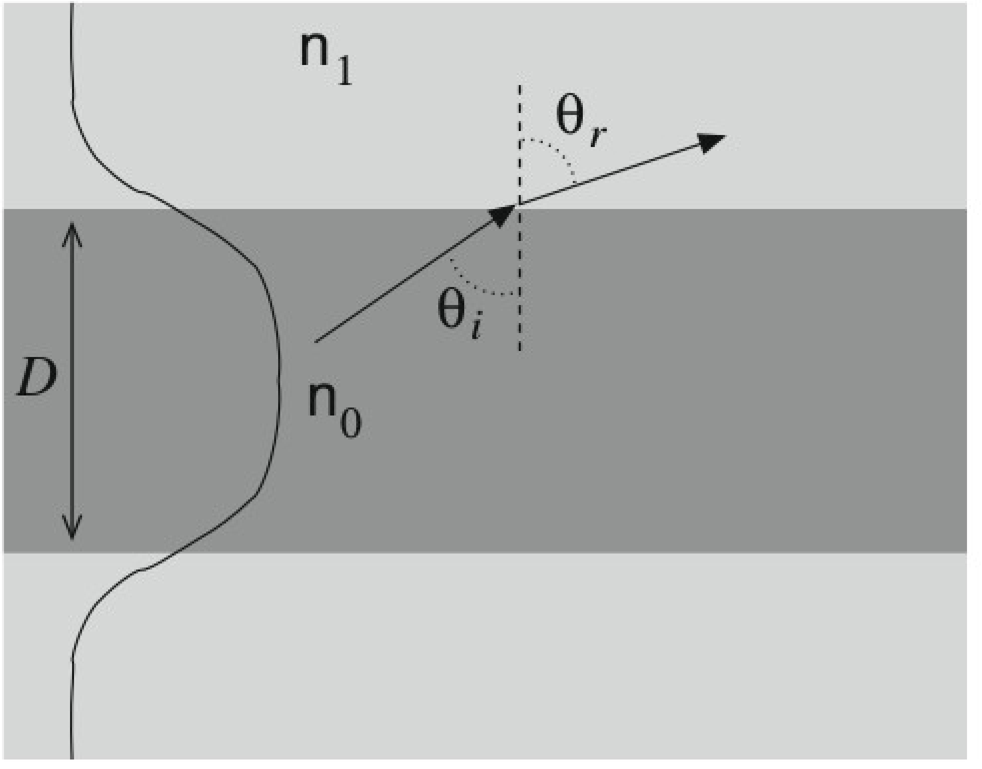
Note that the Rayleigh is usually shorter than L_{acc}. This mean that the laser is diffract before the ideal length.

Self focus

$$n^2 = 1 - \frac{\omega_p^2}{\gamma \omega^2}$$ Nonlinear refractive index

$$\gamma = \left(1 + \frac{a_0^2}{2}\right)^{\frac{1}{2}} \qquad a_o \equiv \frac{eA_0}{m_e c^2}$$

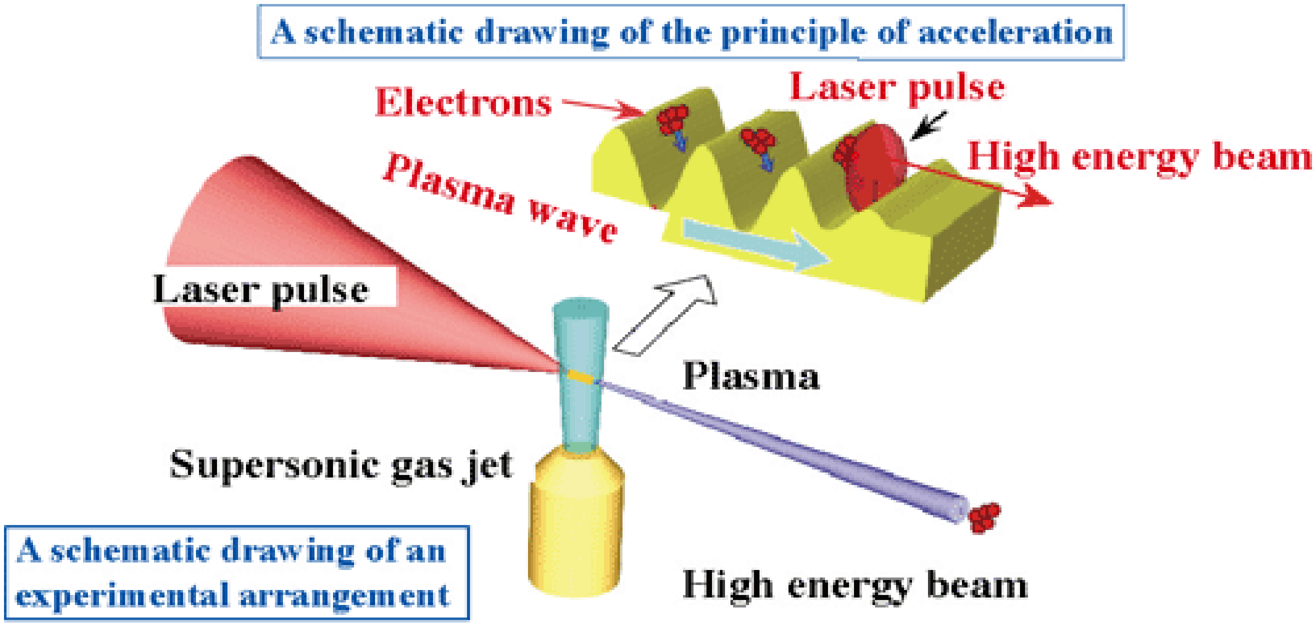
$$\frac{\sin\theta_r}{\sin\theta_i} = \frac{n_0}{n_1} > 1$$

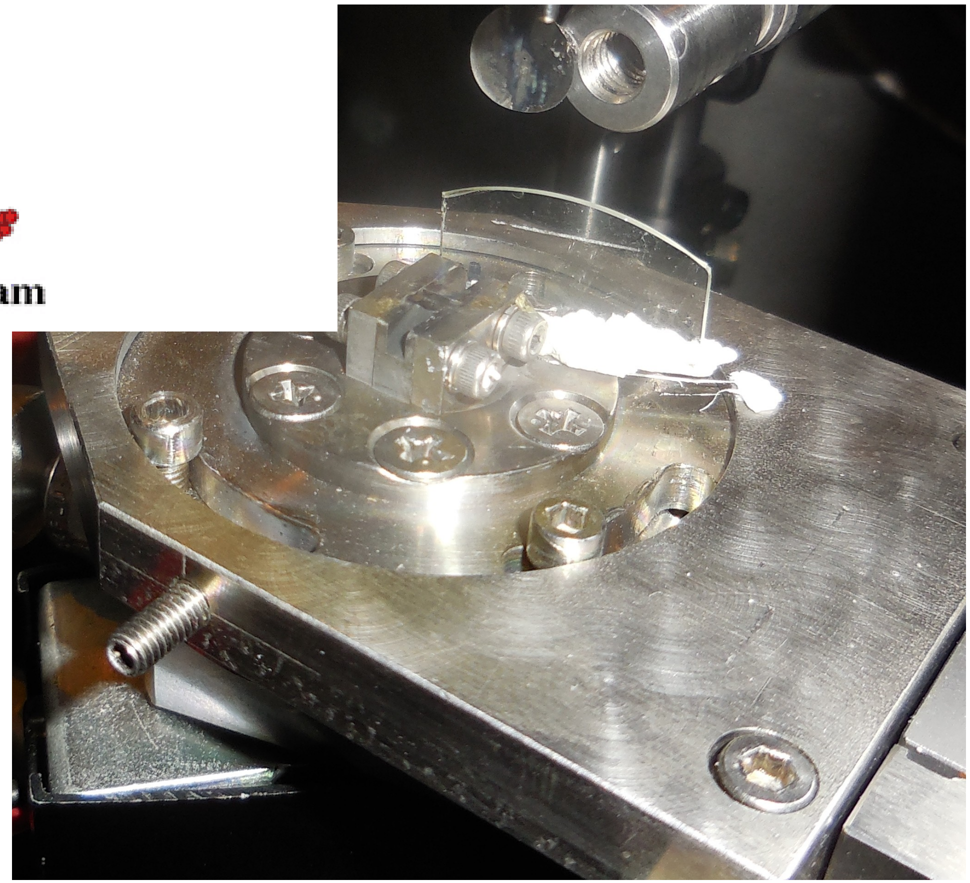


Using Snell's law we find:

$$\frac{n_0}{n_1} = \frac{1}{\sin\theta_i} = \frac{1}{\left(1 - \left(\frac{\lambda}{D}\right)^2\right)^{\frac{1}{2}}} \quad \rightarrow \quad P_c = 43\, GW \frac{n_c}{n_e}$$

Experimental setup

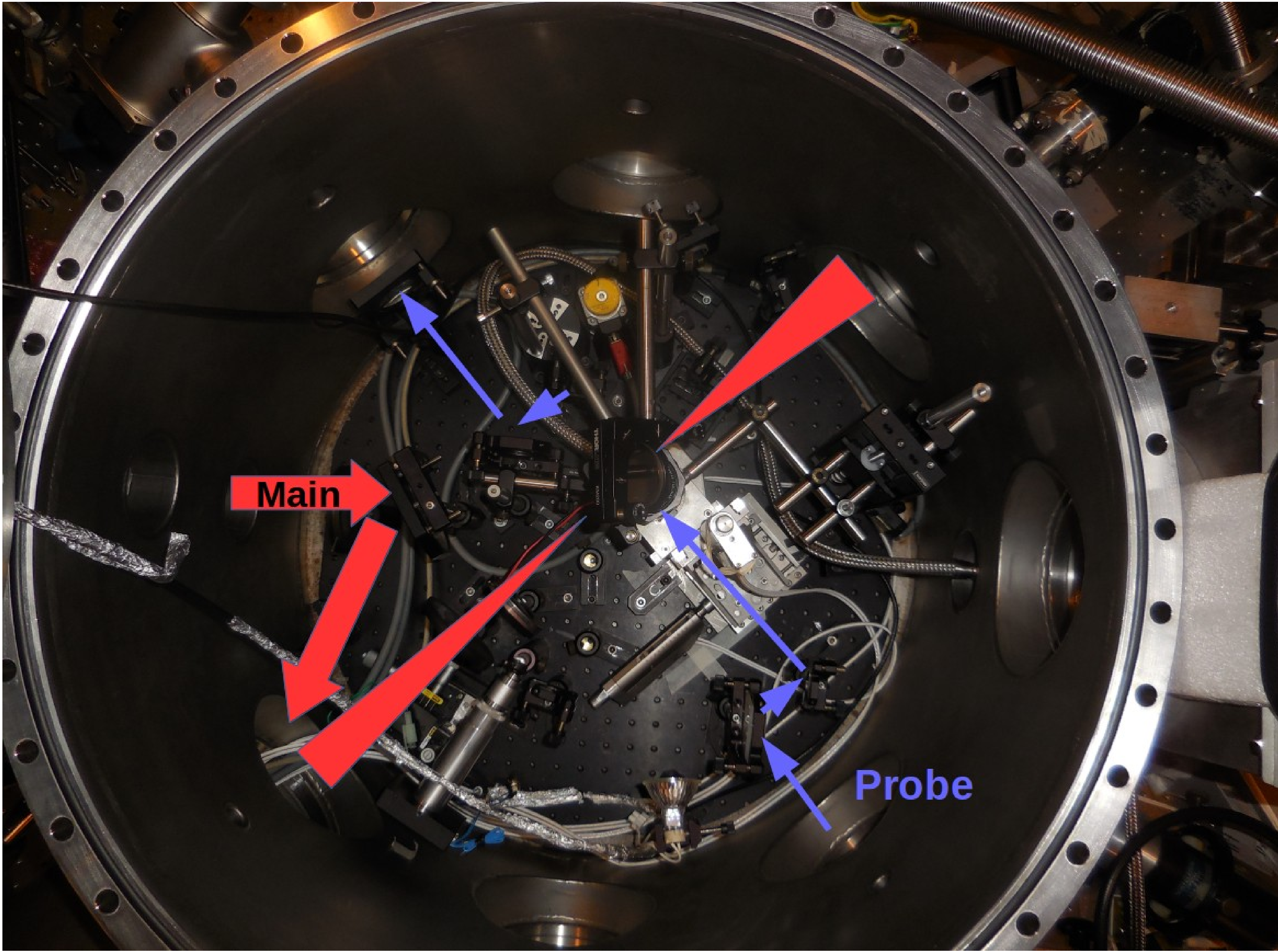




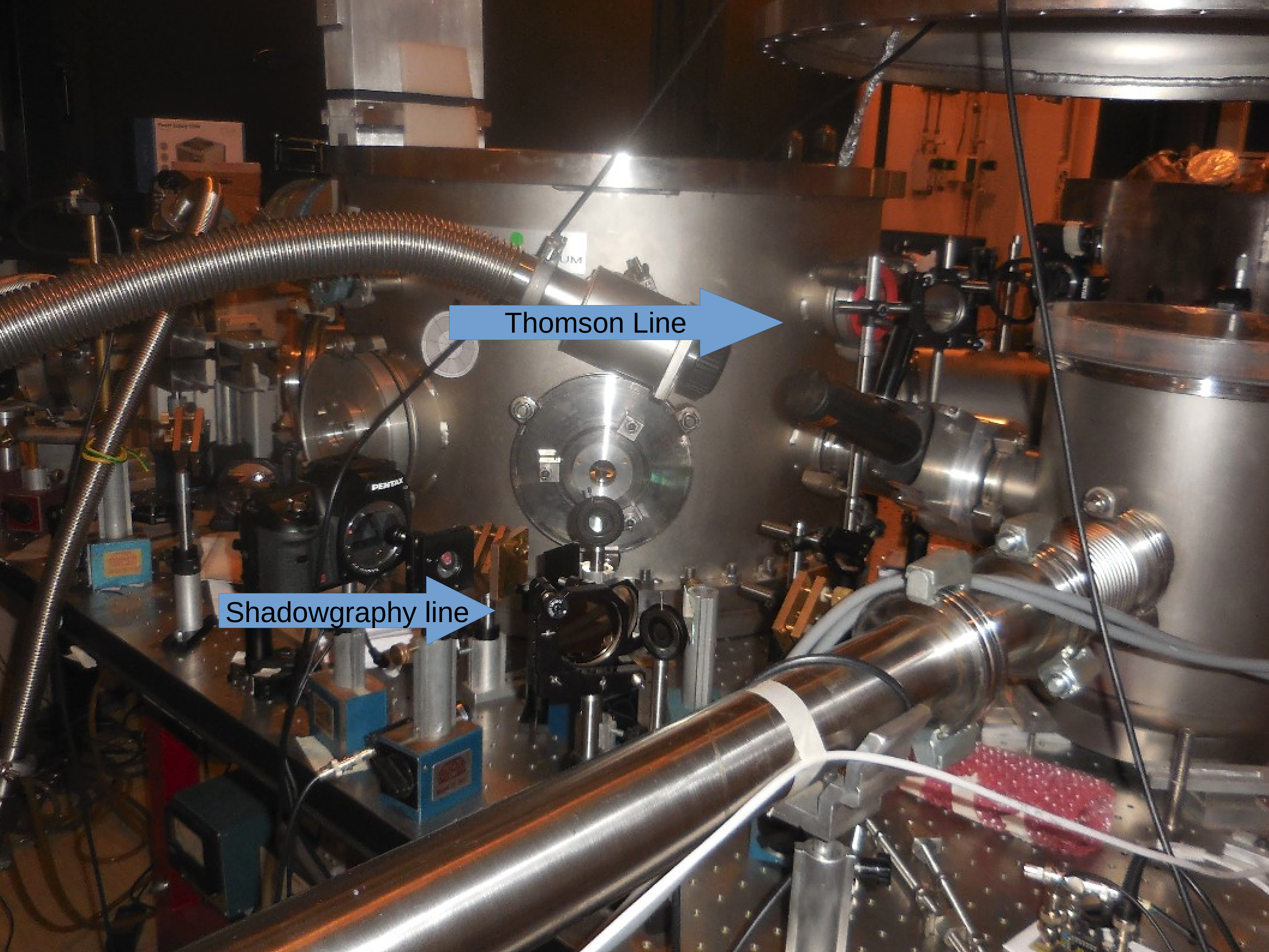
- >400mj on target
- 40 fs duration pulse,
- 800nm Wavelength
- 10 TW Laser
- I=$2x10^{18}$ W/cm^2
- Waist: 8.5 µm

Gas: He, N, Ar, and mixtures

Pressure: 2-50bar


Main
Probe


f/10 OAP

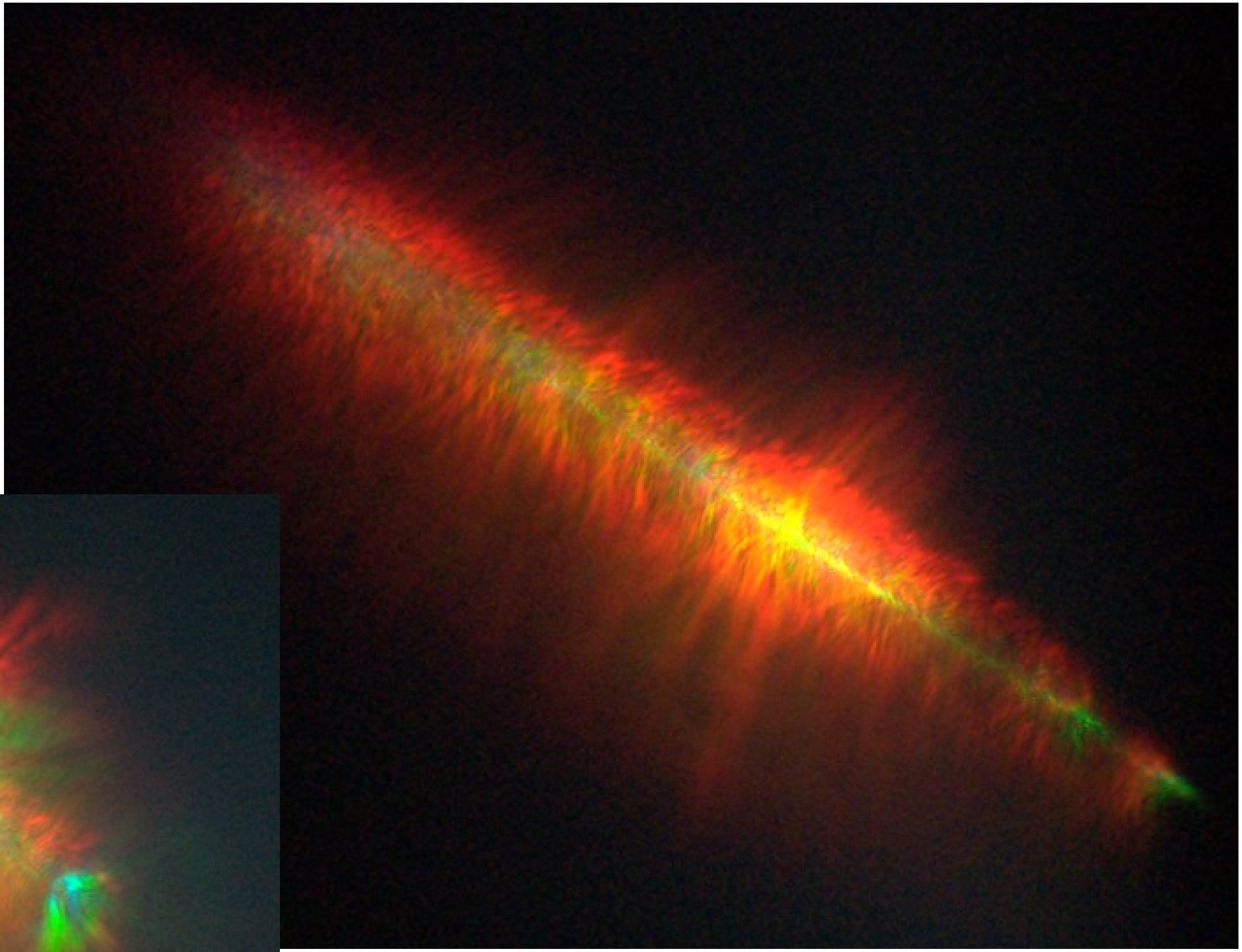

Thomson Line
Shadowgraphy line

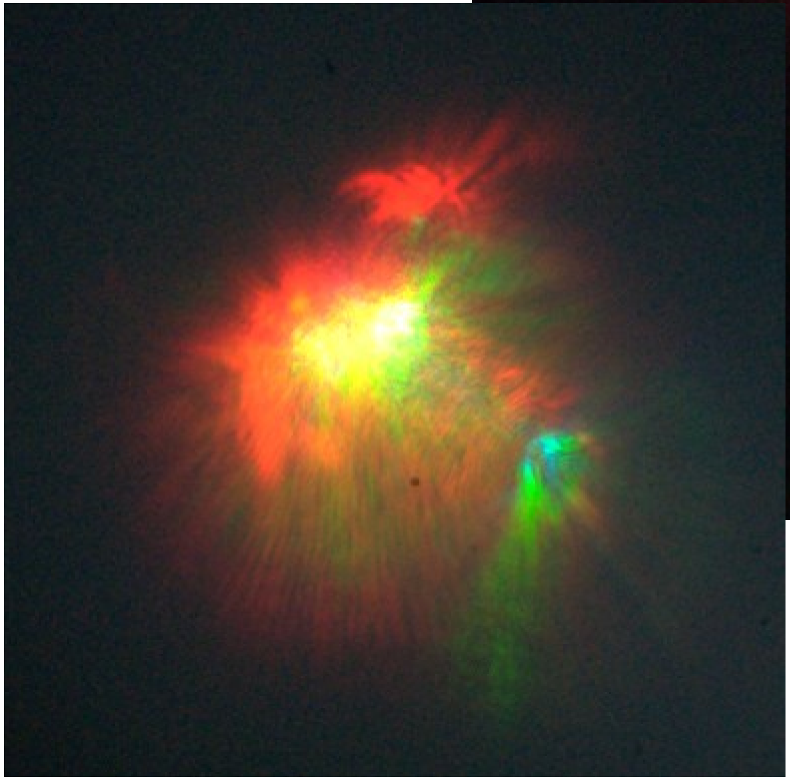
Self-focus evidence

f/5 parabola

400mj on target

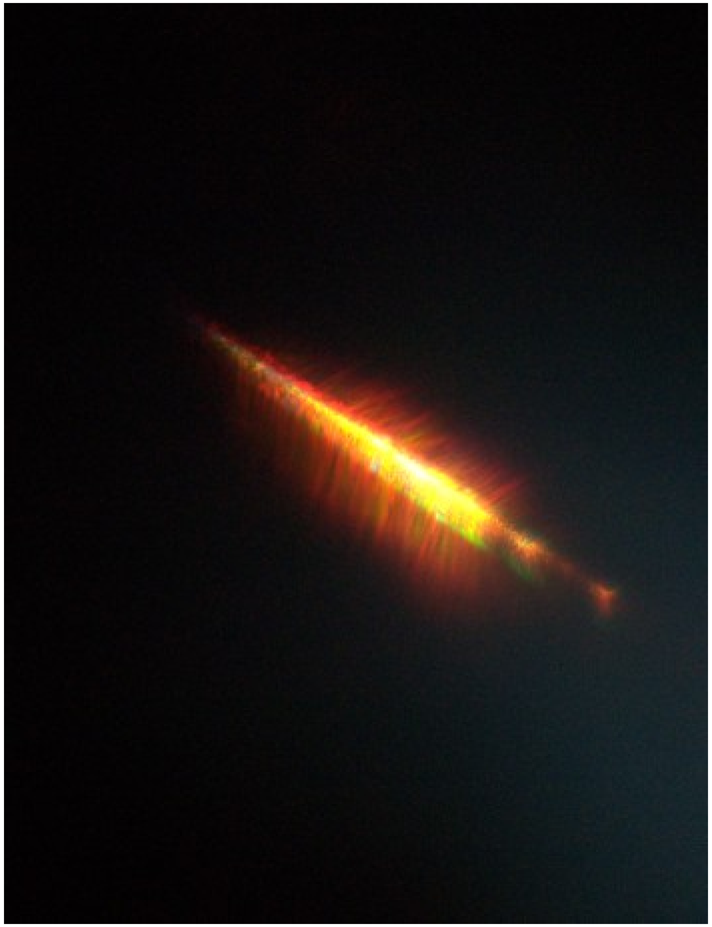
N_2 30bar

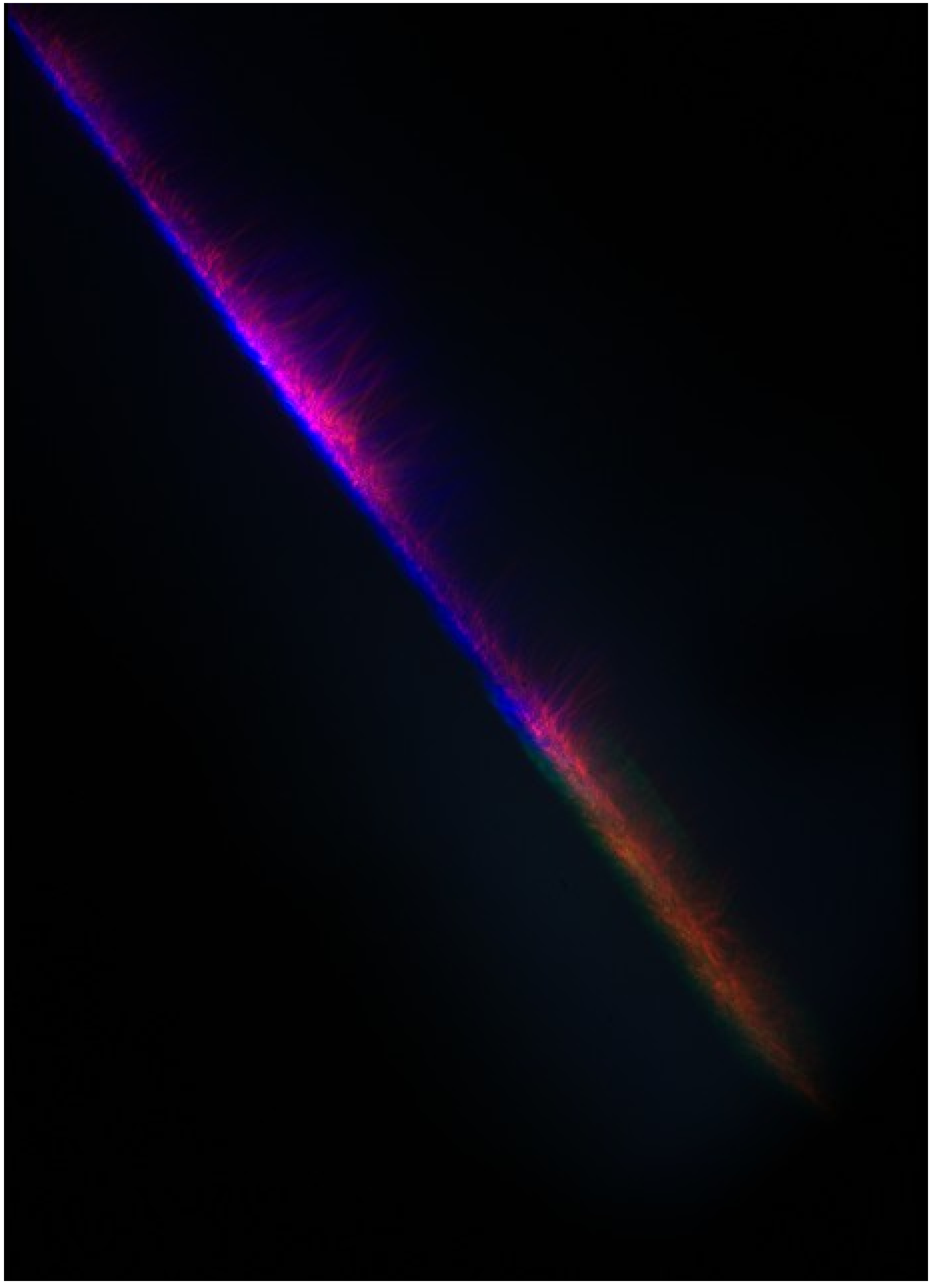




Two shot with the same laser/gas/nozzle condition

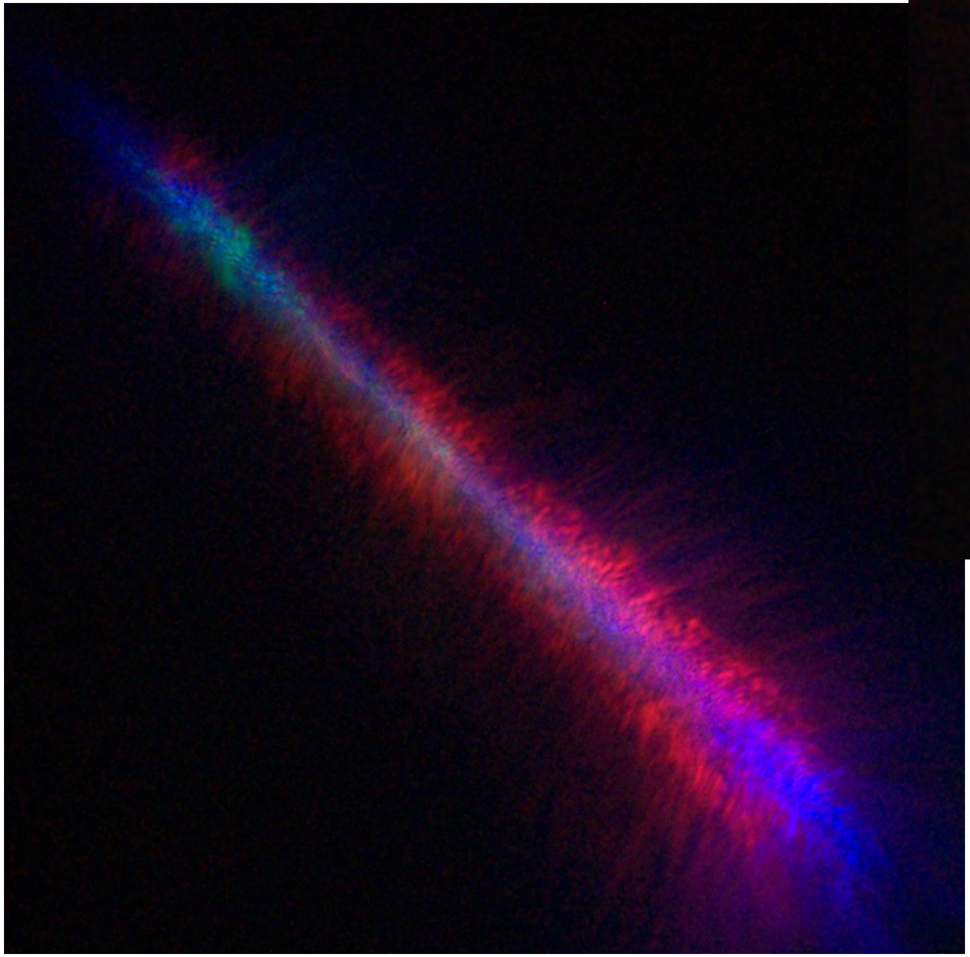
f/5 vs *f*/10 parabola

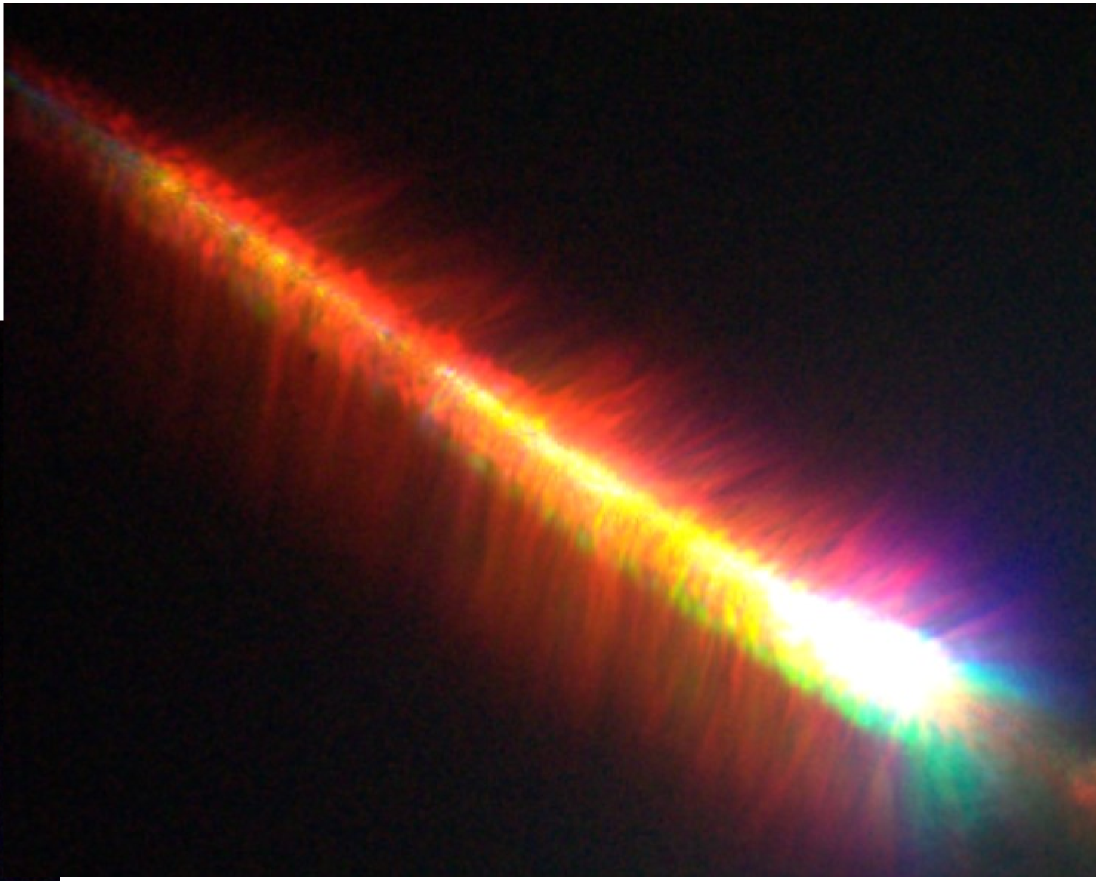




Helium vs Nitrogen

Nitrogen



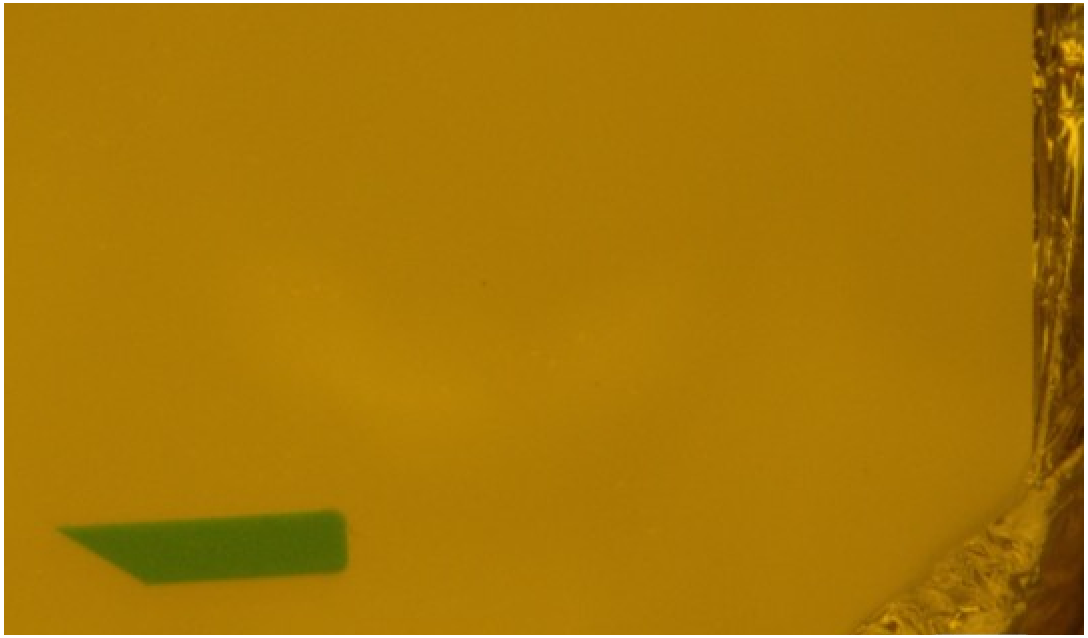


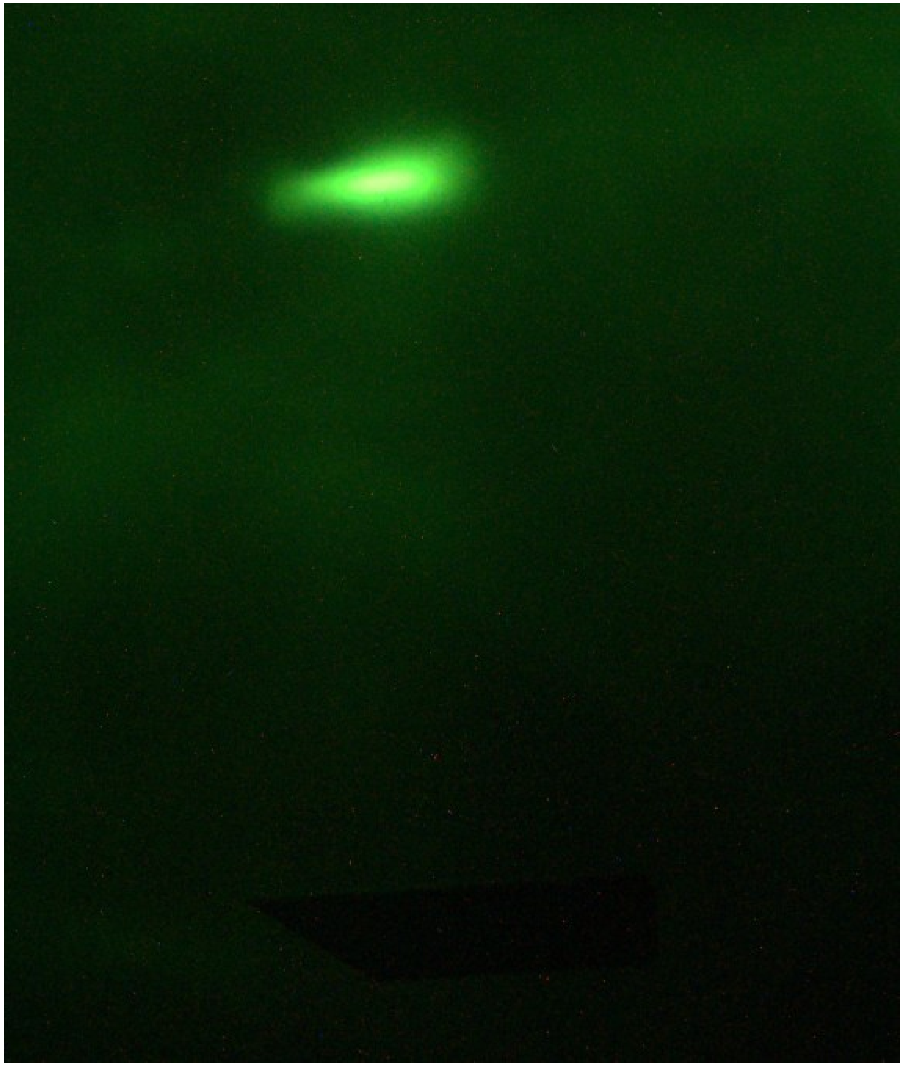
Helium

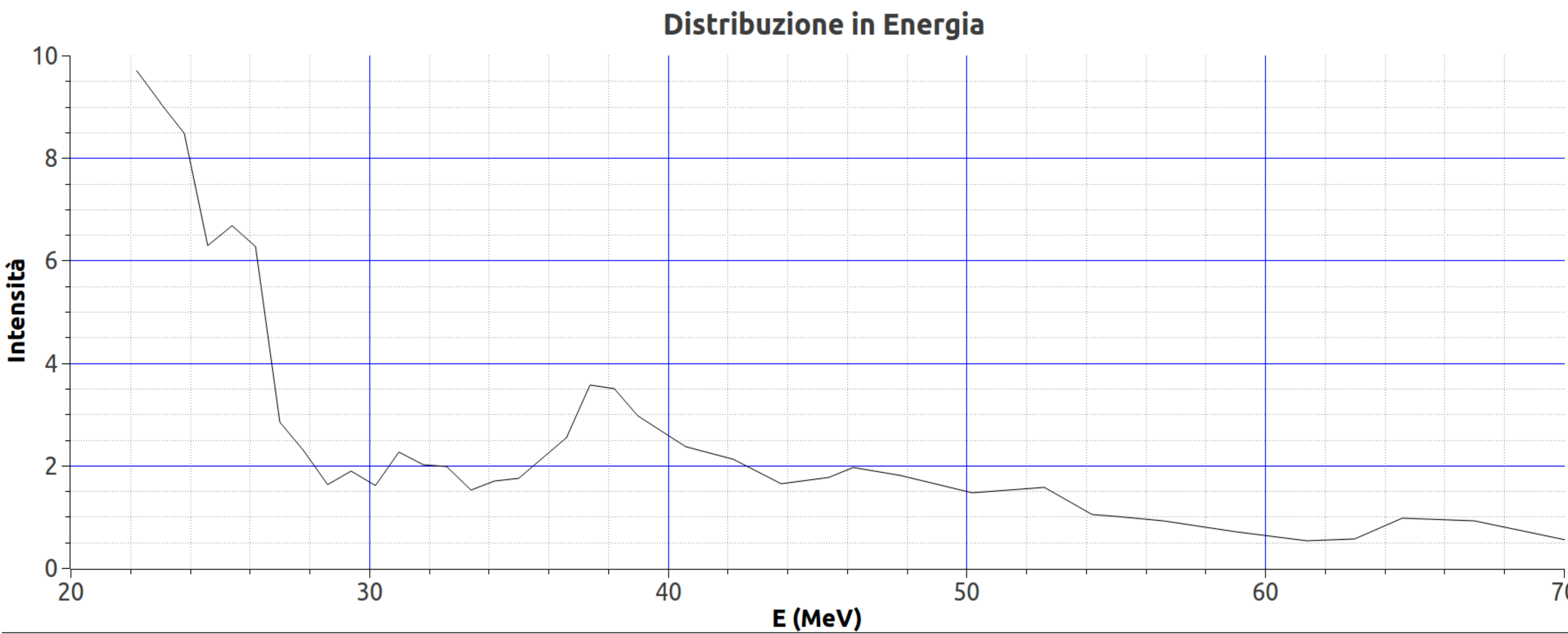
Electron energy

4-5 mrad electron bunch

LANEX scintillator screen






Distribuzione in Energia
Intensità
E (MeV)
0
2
4
6
8
10
20
30
40
50
60
70

Conclusion

- With respect to the standard RF-linacs, the **accelerating distances** of relativistic electron beams is **impressively small** (in principle, 1000 time shorter).
- High-power laser pulses with a suitable femtosecond duration have been develop in **many laboratory** (table-top)
- Plasma based particle acceleration is now regarded as a promising way to **extend performance** of existing accelerators
- Maximum electric field is **not limited** by the breakdown of the walls of the structure(like in a conventional LINAC accelerator)
- Electron bunch is **easy to modify** by changing gas/pressure/nozzle position ecc.
- Possible use of LWFA as a source of high energy particles for **radiobiology** and **radiation therapy**.